Advertising to Status-Conscious Consumers

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Abstract

This paper explores how informative advertising can influence brand image when consumers value social status. It considers a setting where consumers differ in their wealth, which is unobservable, and where a monopolist produces a conspicuous good that allows consumers to signal their wealth through their purchases. The firm chooses price and advertising, and a consumer can only buy the good if he receives an ad. I show that in this setting, informative advertising has effects often associated with persuasion, increasing willingness to pay for the conspicuous good by increasing the stigma associated with not buying. Advertising’s maximum impact on willingness to pay is decreasing with the good’s exclusivity, its impact can be discontinuous in the advertising level, and restricting advertising may be welfare improving. Under duopoly, a firm’s advertising may also increase its rival’s profits.

1. Introduction

Consumers are often influenced not only by the intrinsic quality of what they buy, but also by the image they project about themselves through buying. In this sense, the brand image associated with a particular product will often matter. Different brands can be associated with different personalities, such as sincerity, excitement, and confidence (Aaker, 1997). Luxury brands can also project an image of exclusivity, sophistication and prestige (Kapferer and Bastien, 2009).

Brand image depends in part on a firm’s marketing efforts, in particular on its choice of advertising. Meenaghan (1995) argues that advertising creates imagery, adds symbolic value, and may even graft a particular brand image to a particular product. But it is widely recognized that brand image also depends on a product’s use by consumers. For example, one reason luxury brands project a certain image is that

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only certain types of consumers buy them. Buehler and Halbheer (2011) make a similar distinction between active and passive branding as two channels through which brand image is determined.

This raises the issue of the relative importance of these two channels and how one may interact with the other. That is, how do firm communication and consumer behavior together determine brand image, and how do they influence one another? Keller and Lehmann (2006) and Kuksov et al. (2012) point to this issue as a priority for future research, in particular for socially conspicuous goods whose brand image depends closely on who it is that buys.\textsuperscript{1} This paper aims to take a step in this direction.

I consider a monopolist who advertises and sells a conspicuous good to a market of status-conscious consumers, whose willingness to pay depends on the good’s image. The image arises endogenously and depends on the interplay between purchase behavior and advertising, which I assume to be informative rather than persuasive.

More specifically, I introduce informative advertising into the model of conspicuous consumption by Corneo and Jeanne (1997). Consumers value social status and differ in their wealth. The conspicuous good is observable but wealth is not, so consumers may be able to signal their wealth through their purchases. The firm chooses price and the level of advertising, where ads are sent randomly across consumers, who can only buy the conspicuous good if they receive an ad. In the main analysis, advertising simply informs consumers of the existence of the good, allowing them to buy. In a subsequent section, I assume advertising also allows consumers to recognize the good, making it effectively conspicuous.

I show that in this setting, the firm’s use of advertising can increase consumers’ willingness to pay for the conspicuous good. In the main analysis, advertising affects willingness to pay by changing the composition of the group of consumers who do not buy. Consumers who don’t buy the conspicuous good do so for one of two reasons: either they are relatively poor, or they simply don’t receive an ad. Increased advertising changes the image associated with not buying by decreasing the size of the latter group, so consumers who don’t buy are more closely identified as poor. In this way, informative advertising affects willingness to pay by increasing the stigma associated with not buying.

This mechanism also generates a number of other predictions on the effects of advertising. The maximum extent to which advertising can increase willingness to pay varies monotonically with the good’s exclusivity, and the impact of advertising can be discontinuous in the advertising level. High levels of advertising promote what Corneo and Jeanne (1997) term conformist over snobbish behavior, where the conspicuous good become more attractive when a broader range of consumers buy, which is necessary for demand to be

\textsuperscript{1}In the same spirit, Buehler and Halbheer (2011) write that an interaction between persuasive advertising and consumer social attitudes seems natural, but has been largely ignored in the literature.
upwards sloping. I also show that limiting the amount of advertising can be welfare improving.

I then show that advertising has an additional impact on willingness to pay if it promotes recognition of the conspicuous good. Widespread recognition allows observers to distinguish between consumers who buy and consumers who don’t, increasing the expected image difference between the two. This effect also leads to a novel prediction regarding the firm’s use of advertising when targeting is possible. In the main analysis, the firm would like to target ads precisely on potential demand, to inform consumers who want to buy at the lowest cost. But when advertising promotes recognition, the firm may instead advertise broadly to increase the strength of the signal that consumers send by buying. Finally, I extend the main analysis to duopoly and identify conditions under which a firm’s advertising can increase its rival’s profits.

These results show that in a setting with consumption externalities, informative advertising can have affects more often associated with persuasion. Economists often distinguish between informative advertising, which transmits information about price, product existence or characteristics, and persuasive advertising, which directly provides utility or changes consumer preferences.\(^2\) In this setting, advertising affects willingness to pay even though it is purely informative. The key point is that consumers who receive ads are willing to pay more when they know that many others receive ads as well. Their incentive to buy depends on who else buys the conspicuous good, and on who else can recognize it.

This paper follows Corneo and Jeanne (1997) and others in adopting a signaling approaches to conspicuous consumption, (see, e.g., Bernheim (1994), Ireland (1994), Pesendorfer (1995) and Bagwell and Bernheim (1996)). Related approaches assume social status depends directly on a consumer’s relative position in the consumption distribution (see, e.g., Frank (1985) and Hopkins and Kornienko (2004)), or that consumers are either snobbish or conformist types who experience a direct network effect that depends on quantity sold (see, e.g., Amaldoss and Jain (2005)).

A number of papers have examined the impact of advertising in markets with network externalities. Bagwell and Ramey (1994), Pastine and Pastine (2002), and Clark and Horstmann (2005) all show that advertising can then increase demand by coordinating consumer expectations. This advertising does not transmit information, and is instead just a means of burning money. Sahuguet (2011) does look at how repeated, informative advertising can affect demand in the spirit of Chwe (2001), but does not examine firm incentives. Moreover, consumption externalities are assumed to be positive, whereas externalities here depend on the identity of consumers who buy.

Other recent papers look specifically at advertising to status-conscious consumers. Buehler and Halbheer (2011) examine how brand image can depend on both advertising and consumer purchases, but do not look

\(^2\)See Bagwell (2007) for a thorough review of the literature on the economics of advertising.
at the interaction between the two. Buehler and Halbheer (2012) also show persuasive advertising that
directly affects demand can also indirectly affect the willingness to pay of consumers with social attitudes.
Both papers follow Amaldoss and Jain (2005) in assuming conformist or snobbist types, whose distribution
only affects demand through the average conformity in the population. Other papers instead consider the
relationship between firm communication and consumer signaling. Yoganarasimhan (2012) shows a firm
may withhold product information to ensure that its purchase remains a signal of good taste, and Kuksov
et al. (2012) show that withholding information about aggregate demand may encourage consumers to
communicate between themselves. None of this work considers advertising that is classically informative:
ads that simply inform consumers of a good’s existence and price, allowing them to buy.

The idea that advertising can promote recognition is similar to Krahmer (2006), who shows that allowing
social contacts to recognize particular brands can help consumers signal through their purchases. However,
all consumers are fully informed in his setting, and there is no issue of targeting. The duopoly results here
also bear resemblance to Kuksov and Xie (2010), who show a firm’s cost reductions can increase a rival’s
profits, but who do not consider advertising.

The rest of the paper is organized as follows. Section 2 first presents the model. Section 3 contains
the main analysis under monopoly, Section 4 considers advertising’s impact on recognition, and Section 5
extends the main analysis to duopoly. Section 6 then concludes, and is followed by the appendix.

2. The Model

A monopolist produces a conspicuous good of fixed quality, at constant marginal cost normalized to zero. It
faces a unit mass of consumers who each have unit demand.

Consumers are ordered according to their rank in the wealth distribution. Wealth \( w \) is continuously
distributed with CDF \( F \) and pdf \( f \) on interval \( W \subset \mathbb{R}^+ \), where the rank of a consumer with wealth \( w \) is
\( r = 1 - F(w) \).

Consumers have identical preferences and they derive utility from two sources: status utility from the
conspicuous good and intrinsic utility from a numeraire good. The utility of consumer \( r \) is

\[
U_r = u(c_r) + v_r,
\]

where \( c_r \in \mathbb{R}_{\geq 0} \) is consumption of the numeraire good, and \( v_r \) is his status utility depending on whether
or not he buys the conspicuous good, to be explained below. I assume \( u(c) \) is continuously differentiable
with \( u'(c) > 0 \) and \( u''(c) < 0 \). Whether a consumer buys the conspicuous good is observable, but wealth
and consumption of the numeraire good are not.

All consumers can purchase the numeraire good, but a consumer can only purchase the conspicuous good if he receives an ad. Advertising is purely informative with ads sent randomly across all consumers. The firm chooses advertising level \( \phi \in [0,1] \) at cost \( KA(\phi) \), where \( \phi \) is the fraction of consumers who receive an ad and \( K \) is a shift parameter associated with the advertising technology. Fixing advertising at \( \phi = 1 \) yields the original model of Corneo and Jeanne (1997). I assume \( A(0) = 0, A'(0) = 0, A'(\phi) > 0 \) for all \( \phi > 0 \), and \( A''(\phi) > 0 \). The firm also chooses price \( p \).

Let \( v = s_0 \) denote a consumer’s status utility if he buys a unit of the conspicuous good and \( v = s_1 \) if he does not. Define

\[
s_0 = \mathbb{E} \left[ \int_0^1 a(r) b(r) 1_r dr \right].
\]

(1)

where \( a(r) \) is the status utility from being identified precisely as rank \( r \) in the wealth distribution, and \( b(r) \) and \( 1_r \) are both indicator functions. Let \( b(r) = 1 \) if and only if consumer \( r \) demands a unit of the conspicuous good according to his equilibrium strategy, given \((p, \phi)\), and \( 1_r = 1 \) if and only if consumer \( r \) receives an ad. I assume \( a'(r) < 0 \), so status utility is increasing in perceived wealth. As in Corneo and Jeanne (1997), the status utility from buying is the weighted average of rank utility \( a(r) \) taken over all consumers \( r \) who are expected to buy at price \( p \), but now also taking into account random advertising at level \( \phi \). Similarly, define

\[
s_1 = \mathbb{E} \left[ \int_0^1 a(r)(1 - b(r) 1_r) dr \right].
\]

(2)

which is the weighted average of \( a(r) \), taken over all consumers \( r \) who are not expected to buy, given \((p, \phi)\).

The firm first chooses price and advertising level, which are observed by all consumers who receive an ad. Consumers then simultaneously make a purchase decision. The firm’s strategy is therefore a pair \((p, \phi)\), and the strategy of consumer \( r \) is a choice of \((c_r, b_r)\) if he receives an ad, for each pair \((p, \phi)\), and of \( c_r \) if he does not. I look for a Bayes-Nash equilibrium where the firm maximizes expected profits, given the strategies of consumers, each consumer \( r \) maximizes expected utility for each \((p, \phi)\), given the budget constraint \( c_r + pb_r \leq w_r \), the strategies of other consumers, and beliefs about their type, and where beliefs follow from consumer equilibrium strategies using Bayes’ Rule whenever possible.

By definition, \( s_0 \) and \( s_1 \) are consistent with equilibrium actions, as well as any unilateral deviation from equilibrium by the firm or a consumer. An interpretation is that each consumer is randomly matched with a
social contact after his purchase, where the consumer’s status utility follows from the contact’s beliefs about his type.\textsuperscript{3}

3. Analysis

Define the signaling value of the conspicuous good as 
\[ S = s_0 - s_1, \]
the difference in status utility between buying and not buying. For a given signaling value \( S > 0 \), consumer \( r \) with wealth \( w_r \) is willing to buy at price \( p \) if

\[ u(w_r - p) + S \geq u(w_r). \]

His willingness to pay \( V(r, S) \) is the maximum of his wealth, \( w_r \), and the price \( p \) for which \( S = u(w_r) - u(w_r - p) \), so

\[ V(r, S) = \begin{cases} 
  w_r - u^{-1}(u(w_r) - S), & S < u(w_r) - u(0) \\
  w_r, & S \geq u(w_r) - u(0)
\end{cases} \tag{3} \]

Denote by \( V_i \) the derivative of (3) with respect to its \( i \)th argument, \( i \in \{1, 2\} \). Then \( u'(w) > 0 \) and \( u''(w) < 0 \) imply \( V_1 < 0, V_2 \geq 0, V_{12} \leq 0, V_{11} \leq 0 \) and \( V_{22} \leq 0 \), where these inequalities are all strict if \( V(r, S) < w_r \). Willingness to pay is increasing in wealth and in the signaling value at a decreasing rate, and wealthy consumers are willing to pay more for a marginal increase in signaling value.

By \( V_1 < 0 \), consumers will demand the conspicuous good if and only if their rank is below a certain cut-off. For given advertising \( \phi \) and price \( p \), there exists a cut-off \( r_0 \in [0, 1] \) such that all consumers \( r \in [0, r_0] \) demand a unit of the conspicuous good and all consumers \( r \in (r_0, 1] \) do not. I will often think of the firm choosing \( \phi \) and \( r_0 \) rather than \( \phi \) and \( p \).\textsuperscript{4}

I now show how the signaling value of the conspicuous good depends on \( r_0 \) and \( \phi \). Given advertising \( \phi \) and cut-off \( r_0 \), a mass \( \phi r_0 \) of consumers buy the conspicuous good. These consumers have rank randomly drawn from a uniform distribution on \([0, r_0]\). From (1), the status utility from buying is

\[ s_0 = \int_0^{r_0} a(r) dr \tag{4} \]

for any \( r_0 > 0 \). Define \( s_0(r_0 = 0) = a(0) \), which is the limit of (4) as \( r_0 \) tends to zero.

\textsuperscript{3}With this interpretation, the social contact knows the wealth distribution and consumer equilibrium strategies, and observes both \((p, \phi)\) and whether the consumer buys the conspicuous good.

\textsuperscript{4}There will be a unique \( r_0 \) consistent with \((p, \phi)\) if demand is downwards sloping. This will be the case if consumption is not be too conformist, as defined below in Definition 1.
A mass \((1 - \phi)r_0\) of consumers demand the conspicuous good but cannot buy because they don’t receive an ad. These consumers also have rank randomly drawn from a uniform distribution on \([0, r_0]\). A mass \(1 - r_0\) of consumers do not buy because they don’t demand the good, with rank randomly drawn from a uniform distribution on \((r_0, 1]\). From (2), the status utility from not buying is

\[
s_1 = \frac{(1 - \phi) \int_0^{r_0} a(r) dr + \int_{r_0}^1 a(r) dr}{(1 - \phi)r_0 + (1 - r_0)}
\]

for any \((r_0, \phi) \neq (1, 1)\). Define \(s_1(r_0 = \phi = 1) = a(1)\), which is the limit of (5) evaluated at \(\phi = 1\), as \(r_0\) tends to 1.

The signaling value is therefore

\[
S(r_0, \phi) = s_0(r_0) - s_1(r_0, \phi),
\]

with \(s_0(r_0)\) given by (4) and \(s_1(r_0, \phi)\) given by (5). The signaling value is always positive, \(S(r_0, \phi) > 0\) for all \(r_0 \in [0, 1]\) and \(\phi > 0\), since consumers who buy have lower rank on average than consumers who don’t and status is decreasing in perceived rank.

For any particular \(p\) and \(\phi\), the resulting cut-off \(r_0\) is the solution to \(p = V(r, S)\) from (3) and \(S = s_0(r) - s_1(r, \phi)\) from (6). Quantity sold is \(\phi r_0\), where I can write

\[
r_0 = D(S(r_0, \phi), p),
\]

with \(\frac{\partial D}{\partial S} > 0\) and \(\frac{\partial D}{\partial p} < 0\). Put another way, for any particular \(r_0\) and \(\phi\), the willingness to pay of consumer \(r\) is \(V(r, S(r_0, \phi))\), and the firm can charge a price \(p = V(r_0, S(r_0, \phi))\).

I interpret the cut off \(r_0\) as a measure of exclusivity, where a decrease in \(r_0\) means that the conspicuous good becomes more exclusive as only a smaller range of wealthy consumers remain willing to buy. Following Corneo and Jeanne (1997), I make the following definition:

**Definition 1.** For a given \(r_0\) and \(\phi\), consumption is conformist if \(S_1(r_0, \phi) > 0\), so if the signaling value of the conspicuous good is decreasing in exclusivity. Consumption is snobbish if \(S_1(r_0, \phi) < 0\), so if the signaling value is increasing in exclusivity.

An interpretation is that in a market characterized by conformism, demand for the conspicuous good is driven by consumers’ fear of being identified as poor, while in a market characterized by snobbism, demand is driven by their hope of being identified as rich.

To understand the impact of advertising on willingness to pay, I compare the signaling value \(S(r_0, \phi)\) with the signaling value from Corneo and Jeanne, \(S_B(r_0) \equiv S(r_0, 1)\), which is independent of \(\phi\). \(S_B(r_0)\) gives
the signaling value if all consumers were able to buy the conspicuous good whether or not they received an ad, or if $K$ were sufficiently small for the firm to advertise to all consumers.

In the baseline, the status utility from buying and from not buying are both decreasing in the cut-off rank. The only reason not to buy in the baseline is having low wealth. An increase in exclusivity means the least wealthy consumers who bought now choose not to, which increases the average wealth of both groups. Whether consumption is conformist or snobbish just depends on which of $s_0$ or $s_1$ increases more quickly with $r_0$, which in turn depends only on the shape of the rank utility function $a(r)$.

The following figures illustrate the baseline signaling value when rank utility is quadratic. They plot $s_0$ and $s_1$ as a function of $r_0$, where the signaling value is the vertical distance between the two curves. In the first figure, rank utility is convex so that consumption is snobbish,

![Figure 1](image1)

while in the second figure, rank utility is concave so that consumption is conformist,

![Figure 2](image2)
I begin by examining how advertising affects the signaling value of the conspicuous good. I also describe
how these changes in the signaling value translate into changes in willingness to pay. Recall $V_2 \geq 0$, so
willingness to pay is increasing in signaling value, and is strictly increasing for all consumers for whom the
budget constraint does not bind.

Before starting, it is useful to address a few issues of interpretation. I will often consider how the
signaling value $S(r_0, \phi)$ varies with $\phi$, keeping $r_0$ constant. However, the firm does not directly choose $r_0$,
which instead results from the equilibrium behavior of consumers, given $p$ and $\phi$. A change in $\phi$ that affects
willingness to pay will cause consumers to change their purchasing behavior, thus causing $r_0$ to change as
well.

To help clarify these issues, suppose the firm sets $p$ and $\phi$ with corresponding $r_0$, and consider a marginal
increase in advertising. We can decompose the resulting change in signaling value and willingness to pay
into partial and total effects.

If the change in advertising did not affect willingness to pay, then the same consumers would demand
the good as before, leaving $r_0$ unchanged. However, sales still increase because newly informed consumers
are now able to buy, which in turn affects the signaling value. The resulting partial effect of advertising on
signaling value is $\frac{\partial}{\partial \phi} S(r_0, \phi)$, and the partial effect on willingness to pay is $V_2 \frac{\partial}{\partial \phi} S(r_0, \phi)$.

Summing these changes in willingness to pay over all consumers generates a change in aggregate demand,
which has a series of knock-on effects. It again changes the signaling value, willingness to pay, then aggrega-
te demand and so on, eventually resulting in a new equilibrium cut-off $r_0$. The total effect of increased
advertising on the signaling value is then $\frac{d}{d\phi} S(r_0, \phi) = \frac{\partial}{\partial r_0} S(r_0, \phi) \frac{\partial r_0}{\partial \phi} + \frac{\partial}{\partial \phi} S(r_0, \phi)$, and the total effect on
willingness to pay is $V_2 \frac{d}{d\phi} S(r_0, \phi)$.

Here I separate out the partial effect, the direct impact that informing consumers has on the signaling
value and willingness to pay, from the total effect, which includes the knock-on changes that arise as a result.
I later return to this issue to address how changes in advertising cost affect the equilibrium values of $\phi$, $p$ and $r_0$.

**Proposition 1.** For any $\phi < 1$, the signaling value of the conspicuous good is strictly lower than when $\phi = 1$. That is, for all $r_0 \in (0, 1]$,

$$S_B(r_0) - S(r_0, \phi) > 0.$$ 

*Proof.* By definition, write

$$S_B(r_0) - S(r_0, \phi) = s_0(r_0) - s_1(r_0, 1) - (s_0(r_0) - s_1(r_0, \phi)),
$$

$$= s_1(r_0, \phi) - s_1(r_0, 1).$$

Substituting from (5) gives

$$S_B(r_0) - S(r_0, \phi) = \frac{\int_{r_0}^{1} a(r)dr}{1 - r_0} - \frac{(1 - \phi) \int_{0}^{r_0} a(r)dr + \int_{r_0}^{1} a(r)dr}{1 - \phi r_0}.$$

Rearranging yields

$$S_B(r_0) - S(r_0, \phi) = \left(\frac{1 - \phi r_0}{1 - \phi r_0}\left(\frac{\int_{0}^{r_0} a(r)dr}{r_0} - \frac{\int_{r_0}^{1} a(r)dr}{1 - r_0}\right)\right),$$

which is strictly positive since the expression in large brackets is just $S_B(r_0) > 0$. 

An increase in signaling values yields an increase in willingness to pay for all consumers for whom the budget constraint does not bind, since then $V_2 > 0$. That is, when the firm doesn’t advertise the conspicuous good to all consumers, willingness to pay is strictly lower than when it does.

The firm’s choice of advertising increases willingness to pay by changing the stigma associated with not buying the conspicuous good. Increased advertising affects the number of consumers who buy, but not their composition; they are simply those consumers with high wealth. This implies the status utility from buying is independent of advertising. In contrast, consumers who do not buy comprise two distinct groups: poor consumers who don’t want to buy, and wealthy consumers who don’t receive ad. Increased advertising decreases the size of the latter group, so that consumers who don’t buy are more closely identified as being poor.

Since advertising affects willingness to pay, it generally also affects the firm’s optimal price. The numerical simulations at the end of this section show that the optimal price can be either increasing or decreasing in
the level of advertising. This stands in contrast to the standard analysis of informative advertising under monopoly with constant marginal costs, where the optimal price is independent of advertising (see, e.g., Bagwell (2007) p.58).

Proposition 1 shows that for any given advertising level \( \phi \), a sufficiently large increase to \( \phi = 1 \) will always have a positive impact on willingness to pay. The following result shows that the magnitude of this impact depends on the good’s exclusivity.

**Proposition 2.** For any \( \phi < 1 \), the difference in signaling value compared to \( \phi = 1 \) is decreasing in the conspicuous good’s exclusivity. That is,

\[
\frac{\partial}{\partial r_0} (S_B(r_0) - S(r_0, \phi)) > 0.
\]

**Proof.** Substitute in \( S_B(r_0) \) on the right-hand side of (8) and differentiate so that

\[
\frac{\partial}{\partial r_0} (S_B(r_0) - S(r_0, \phi)) = \left( \frac{(1 - \phi)(1 - \phi r_0) + \phi(1 - \phi)r_0}{(1 - \phi r_0)^2} \right) S_B + \left( \frac{(1 - \phi)r_0}{1 - \phi r_0} \right) \frac{d}{dr_0} S_B(r_0).
\]

Simplifying gives

\[
\frac{\partial}{\partial r_0} (S_B(r_0) - S(r_0, \phi)) = \left( \frac{1 - \phi}{(1 - \phi r_0)^2} \right) S_B(r_0) + \left( \frac{(1 - \phi)r_0}{1 - \phi r_0} \right) \frac{d}{dr_0} S_B(r_0),
\]

which is positive if and only if

\[
S_B(r_0) + r_0(1 - \phi r_0) \frac{d}{dr_0} S_B(r_0) > 0. \tag{9}
\]

We know \( S_B(r_0) > 0 \), whereas \( \frac{d}{dr_0} S_B(r_0) \) may be either positive or negative. Hence to show that (9) always holds, it is sufficient to show it holds when \( \phi = 0 \),

\[
S_B(r_0) + r_0 \frac{d}{dr_0} S_B(r_0) = \frac{d}{dr_0} (r_0 S_B(r_0)) > 0.
\]

By the definition of \( S_B(r_0) \), write

\[
\int_0^{r_0} a(r)dr = \frac{r_0}{1 - r_0} \int_{r_0}^1 a(r)dr.
\]

This gives

\[
\frac{d}{dr_0} (r_0 S_B(r_0)) = a(r_0) + \frac{r_0}{1 - r_0} a(r_0) - \frac{1}{(1 - r_0)^2} \int_{r_0}^1 a(r)dr.
\]

\[
= \frac{1}{1 - r_0} \left( a(r_0) - \frac{\int_{r_0}^1 a(r)dr}{1 - r_0} \right),
\]

11
where the expression in large brackets is positive because \( a'(r) < 0 \).

The extent to which a discrete change in advertising can impact the signaling value is largest when the conspicuous good is non-exclusive, so when \( r_0 \) is close to 1. In contrast, the impact is close to zero when the conspicuous good is very exclusive, so when \( r_0 \) is close to zero. Advertising affects the signaling value by increasing the stigma associated with not buying, which can be particularly high when only the poorest consumers are unwilling to buy.

Figure 3 shows the signaling value as a function of \( r_0 \) for \( \phi = 0.7 \) and \( \phi = 1 \), for the same rank utility function as in Figure 1. As required by Proposition 2, the vertical difference between the two curves is decreasing in exclusivity.

Figure 3

\[
\text{Signaling Value, } a(r) = 1.1 - 2r + r^2
\]

This difference in signaling value is precisely equal to the difference in the status utility from not buying, as illustrated in Figure 4.

Figure 4
Recall that for any given level of advertising, exclusivity determines the relative size of the two groups of consumers who do not buy: first, poor consumers who simply don’t want to buy, and second, wealthy consumers who don’t receive an ad. When the good is quite exclusive, so when $r_0$ is close to 0, the first group is much larger than the second, even when advertising levels are low. Increased advertising makes the second group even smaller, but this change has little impact on the average wealth of consumers who do not buy. When the good is very non-exclusive, so when $r_0$ is close to 1, the situation is reversed and advertising can have a large impact.

The extent to which advertising can affect willingness to pay, rather than just the signaling value, is also often decreasing in exclusivity. Since $V_2 > 0$ and $V_{22} < 0$, the impact of moving from $S(r_0, \phi)$ to $S_B(r_0)$ depends on both the magnitude of the change, $S_B(r_0) - S(r_0, \phi)$, and the initial signaling value, $S(r_0, \phi)$. For the impact on willingness to pay to be decreasing in exclusivity, a sufficient condition is that $S_B(r_0) - S(r_0, \phi)$ be increasing in $r_0$, which holds by Proposition 2, and that $S_B(r_0)$ be decreasing in $r_0$. Proposition 6 will show this is the case whenever consumption is snobbish in the baseline or whenever $\phi$ is sufficiently small.

This impact of advertising on demand can be related to Johnson and Myatt (2006)’s notions of real information and hype. In their terminology, real information rotates the demand curve, increasing demand from some consumers and decreasing demand from others, while hype shifts the demand curve out, increasing quantity demanded at every price. Proposition 2 implies that advertising has effects similar to both real information and hype. Increased advertising can effectively rotate the inverse demand curve, but around its intercept on the vertical axis, which increases quantity demanded at every price.

The analysis up until now has considered a discrete change in advertising, comparing willingness to pay when $\phi < 1$ with the baseline when $\phi = 1$. I now consider the impact of increasing advertising on the margin.

**Proposition 3.** Suppose $\phi < 1$. Then for all $r_0 \in (0, 1)$, the marginal impact of advertising on the signaling
value is positive: $\frac{\partial}{\partial \phi} S(r_0, \phi) > 0$. For both $r_0 = 0$ and $r_0 = 1$, the marginal impact of advertising on the signaling value is zero: $\frac{\partial}{\partial \phi} S(0, \phi) = \frac{\partial}{\partial \phi} S(1, \phi) = 0$.

Proof. Substituting in $S_B(r_0)$ on the right-hand side of (8) and rearranging yields

$$S(r_0, \phi) = \left(1 - \frac{r_0}{1 - \phi r_0}\right) S_B(r_0),$$

where $S_B(r_0)$ is independent of $\phi$. This implies

$$\frac{\partial}{\partial \phi} S(r_0, \phi) = \frac{r_0(1 - r_0)}{(1 - \phi r_0)^2} S_B(r_0), \quad (10)$$

which equals to zero for $r_0 = 0$ and $r_0 = 1$, and is strictly positive for all $r_0 \in (0, 1)$.

The marginal impact of advertising is positive, which is in line with Proposition 2. However, while the impact of a sufficiently large change in advertising is decreasing in exclusivity, the impact of a marginal change is non-monotonic. It is small when the good is exclusive but also when it is non-exclusive, so when $r_0$ is close to 1, which is precisely when Proposition 2 shows advertising can have the largest effect on the signaling value.

To reconcile these facts, note from (10) that the marginal impact of advertising on the signaling value is increasing in the advertising level. That is, advertising increases the signaling value of the conspicuous good at an increasing rate. When the good is quite non-exclusive, the impact of a marginal increase in advertising is negligible for most values of $\phi$, but very large when $\phi$ is close to 1. In this sense, a small increase in advertising can have a large impact on willingness to pay when many consumers already buy the good.

This is illustrated in Figure 5, which show the marginal impact of advertising on signaling value as a function of $r_0$, for $\phi = 0.7$ and $\phi = 0.9$.

Figure 5
The starkest case of a small change in advertising having a large effect is when \( r_0 = 1 \), so when all consumers demand the good. The relationship between advertising level and willingness to pay is then discontinuous.

**Proposition 4.** Suppose \( r_0 = 1 \) and \( \phi < 1 \). In the limit as \( \phi \) approaches 1, the difference in signaling value compared with \( \phi = 1 \) remains strictly positive,

\[
\lim_{\phi \to 1} (S_B(1) - S(1, \phi)) > 0.
\]

**Proof.** From (8), write

\[
S_B(r_0) - S(r_0, \phi) = \frac{(1 - \phi)r_0}{1 - \phi r_0} S_B(r_0).
\]

Evaluating at \( r_0 = 1 \) and canceling \((1 - \phi) > 0\) in numerator and denominator gives

\[
S_B(1) - S(1, \phi) = S_B(1) = \int_0^1 a(r)dr - a(1),
\]

which is independent of \( \phi \). It is strictly positive since \( a'(r) < 0 \).

When all consumers demand the good, willingness to pay is independent of advertising for all \( \phi < 1 \). The signaling value is in fact zero, since consumers who don’t buy and consumers who do are all believed to have average wealth. The signaling value then takes a discrete jump at \( \phi = 1 \) as a consumer who doesn’t buy is believed to have the lowest wealth, rank \( r_0 = 1 \). This implies that the firm will never sell to the poorest consumers unless it also advertises to everyone.
The discontinuity does rely on out-of-equilibrium beliefs when $r_0 = \phi = 1$. However, a similar discontinuity will occur as long as a consumer who deviates is believed to have below average wealth. Moreover, the incentive to deviate is strictly decreasing in wealth, so the belief $r_0 = 1$ is reasonable in the spirit of the D1 Criterion (Cho and Kreps, 1987). Any other out-of-equilibrium belief would also lead to an unintuitive discontinuity in the baseline that does not occur in Corneo and Jeanne (1997). The formal point is that the status utility from not buying is not the same when $r_0 = 1$ and $\phi$ tends to 1, as it is when $\phi = 1$ and $r_0$ tends to 1.

I now show that, unlike in the baseline, the stigma associated with not buying can actually increase when the conspicuous good becomes less exclusive.

**Proposition 5.** For any $\phi < 1$, the status utility from not buying is non-monotonic in the exclusivity of the conspicuous good. That is, there exists $\theta^* \in (0, 1)$, such that $\frac{\partial}{\partial r_0} s_1(r_0, \phi) < 0$ for all $r_0 \in [0, \theta^*)$, $\frac{\partial}{\partial r_0} s_1(r_0, \phi) = 0$ for $r_0 = \theta^*$, and $\frac{\partial}{\partial r_0} s_1(r_0, \phi) > 0$ for all $r_0 \in (\theta^*, 1]$. Moreover, $\theta^*$ is increasing in $\phi$.

**Proof.** From (5), write

$$s_1(r_0, \phi) = \frac{(1 - \phi) \int_0^{r_0} a(r) dr + \int_{r_0}^{1} a(r) dr}{1 - \phi r_0}.$$  

Differentiating gives

$$\frac{\partial}{\partial r_0} s_1(r_0, \phi) = \frac{\left((1 - \phi)a(r_0) - a(r_0)\right)\left(1 - \phi r_0\right) + \phi \left((1 - \phi) \int_0^{r_0} a(r) dr + \int_{r_0}^{1} a(r) dr\right)}{(1 - \phi r_0)^2},$$

which is positive if and only if

$$-a(r_0)(1 - \phi r_0) + (1 - \phi) \int_0^{r_0} a(r) dr + \int_{r_0}^{1} a(r) dr \geq 0,$$

or equivalently

$$\int_0^{1} a(r) dr - \phi \int_0^{r_0} a(r) dr - a(r_0)(1 - \phi r_0) \geq 0.$$  

Condition (12) is violated at $r_0 = 0$ since

$$\int_0^{1} a(r) dr - a(0) < 0,$$

and satisfied strictly at $r_0 = 1$ since

$$(1 - \phi) \left(\int_0^{1} a(r) dr - a(1)\right) > 0.$$
in both cases because \(a'(r) < 0\). The left-hand side of (12) is also increasing with \(r_0\), since

\[
\frac{\partial}{\partial r_0} \left( \int_0^1 a(r)dr - \phi \int_0^{r_0} a(r)dr - a(r_0)(1 - \phi r_0) \right) = -a'(r_0)(1 - \phi r_0) > 0.
\]

Hence there exists \(\theta^* \in (0, 1)\) such that (12) is violated for all \(r_0 \in [0, \theta^*]\), satisfied with equality for \(r_0 = \theta^*\), and satisfied strictly for all \(r_0 \in (\theta^*, 1]\).

To show that \(\theta^*\) is increasing in \(\phi\), it is sufficient to show that

\[
\frac{\partial}{\partial \phi} \left( \int_0^1 a(r)dr - \phi \int_0^{r_0} a(r)dr - a(r_0)(1 - \phi r_0) \right) < 0.
\]

This derivative is equal to

\[-\int_0^{r_0} a(r)dr + a(r_0)r_0 = r_0 \left( a(r_0) - \frac{\int_0^{r_0} a(r)dr}{r_0} \right),\]

which is negative since \(a'(r) < 0\).

When \(\phi < 1\), the status utility from not buying need not decrease as the conspicuous good becomes less exclusive. It is still decreasing in exclusivity when \(r_0\) is small, so when the good is quite exclusive, but increasing in exclusivity after \(r_0\) exceeds a threshold value. The status utility from not buying is highest when the good is either very exclusive or very non-exclusive, so when \(r_0 = 0\) or \(r_0 = 1\). This is illustrated by the dotted curve seen earlier in Figure 3.

The intuition is that when the good is exclusive, the majority of consumers who don’t buy simply don’t want to. They have wealth below a certain threshold which is quite high. When the good is non-exclusive, the majority of consumers who don’t buy do want to but do not receive an ad. They still have wealth above a certain threshold, but the threshold is now quite low. In both cases, the average wealth of consumers who don’t buy is close to the average wealth in the population. The average wealth of consumers who buy is higher in the former case than it is in the latter, which explains the difference in willingness to pay.

I now show that whether consumption is conformist or snobbish does not depend only on the rank utility function \(a(r)\), as it does in the baseline. It also depends on the level of advertising.

**Proposition 6.** If consumption is snobbish when \(\phi = 1\), then it is also snobbish for all \(\phi < 1\). That is, at any \(r_0\) for which \(\frac{\partial}{\partial r_0} S_B(r_0) < 0\), then \(\frac{\partial}{\partial r_0} S(r_0, \phi) < 0\) as well.

For any \(\phi < 1\), there are certain values of \(r_0\) for which consumption is snobbish. That is, there exists \(\theta^* \in [0, 1)\) such that \(\frac{\partial}{\partial r_0} S(r_0, \phi) < 0\) for all \([\theta^*, 1]\). Moreover, when \(\phi\) is sufficiently small, \(\theta^* = 0\).
Proof. Proposition 2 shows that 
\[
\frac{d}{dr}S_B(r_0) - \frac{\partial}{\partial r_0}S(r_0, \phi) > 0.
\]
Hence \(\frac{\partial}{\partial r_0}S_B(r_0) < 0\) implies 
\[
\frac{\partial}{\partial r_0}S(r_0, \phi) < \frac{d}{dr_0}S_B < 0.
\]
By the definition of \(S(r_0, \phi)\), write 
\[
\frac{\partial}{\partial r_0}S(r_0, \phi) = \frac{\partial}{\partial r_0}s_0(r_0) - \frac{\partial}{\partial r_0}s_1(r_0, \phi),
\]
where \(\frac{\partial}{\partial r_0}s_0(r_0) < 0\). From Proposition 5, there exists \(\theta^* \in (0, 1)\) such that \(\frac{\partial}{\partial r_0}s_1(r_0, \phi) > 0\) for all \(r_0 \in [\theta^*, 1]\). This implies that for all such \(r_0 \in [\theta^*, 1]\), \(\frac{\partial}{\partial r_0}S(r_0, \phi) < 0\).

By (11), in the limit as \(\phi\) approaches zero, \(\frac{\partial}{\partial r_0}s_1(r_0, \phi)\) approaches zero as well, for all \(r_0\). Since \(\frac{\partial}{\partial r_0}s_0(r_0)\) is strictly negative for all \(r_0\) and independent of \(\phi\), this implies \(\frac{\partial}{\partial r_0}S(r_0, \phi) < 0\) when \(\phi\) is sufficiently small.

High levels of advertising tends to make consumption less snobbish and more conformist. Consumers become more concerned with avoiding the stigma associated with not buying, which is increasing with advertising, rather than enjoying the high status from buying, which is not.

Consumption will be snobbish for all values of \(r_0\) when advertising is sufficiently low, regardless of the shape of the rank utility function \(a(r)\), even if consumption is conformist for all \(r_0\) in the baseline. The reason is that with low levels of advertising, the status utility from not buying varies very little with exclusivity. Since most consumers don’t buy, their average wealth remains close to the average wealth in the population. The average wealth of consumers who do buy does not depend on advertising and is decreasing in exclusivity, so the status incentive to buy is decreasing as well.

Proposition 6 also implies that demand can never be upwards sloping unless the advertising is sufficiently high. Taking the differential of \(r_0 = D(S(r_0, \phi), p)\) from (7) yields 
\[
\frac{dr_0}{dp} = \frac{\partial D}{\partial S} \frac{\partial S}{\partial r_0} dr_0 + \frac{\partial D}{\partial p} dp,
\]
so that 
\[
\frac{dr_0}{dp} = \left( \frac{1}{1 - \frac{\partial D}{\partial S} \frac{\partial S}{\partial r_0}} \right) \frac{\partial D}{\partial p}.
\]
(13)
Demand is locally downwards sloping if, for a fixed advertising level, quantity demanded is decreasing in price: \( \frac{\partial q}{\partial p} < 0 \). By \( \frac{\partial D}{\partial S} > 0 \) and \( \frac{\partial D}{\partial p} < 0 \), a sufficient condition for demand to be downward sloping at price \( p \) and cut-off \( r_0 \) is that \( \frac{\partial S}{\partial r_0} < 0 \), so that consumption is snobbish. It follows immediately from Proposition 6 that demand is everywhere downwards sloping whenever \( \phi \) is sufficiently small. Moreover, a necessary condition for demand to be everywhere upwards sloping is \( \phi = 1 \), since otherwise consumption is snobbish in a neighborhood of \( r_0 = 1 \).

I now address how the firm’s optimal choice of \((p, \phi)\), or equivalently \((r_0, \phi)\), depend on the advertising cost \( K \). The following result shows that the optimal \( \phi \) is decreasing in \( K \), where the proof can be found in the appendix.

**Proposition 7.** The optimal advertising level is strictly positive and decreasing in advertising cost. That is, \( \phi'(K) \leq 0, \phi > 0 \) and \( \lim_{K \to \infty} \phi(K) = 0 \).

Higher values of \( \phi \) can be identified with lower values of \( K \), and vice versa. This allows me to examine how the firm’s optimal price and choice of exclusivity varies with \( \phi \), and interpret this as comparative statics for \( p \) and \( r_0 \) with respect to \( K \).

With this interpretation, under quite general conditions, a reduction in advertising costs causes the firm to make the conspicuous good less exclusive.

**Proposition 8.** Suppose that any one of the following conditions holds: (i) utility is sufficiently close to linear in the numeraire good, \( \frac{d^2 u(i)}{d \epsilon^2} < \epsilon \) for all \( i \geq 2 \), for some \( \epsilon > 0 \) sufficiently small, (ii) consumption is sufficiently snobbish in the baseline, \( f(w(r_0)) \frac{d}{dr_0} S_B(r_0) \leq -u'(w(r_0)) \) for all \( r_0 \in [0,1] \), (iii) consumption is sufficiently conformist in the baseline, \( f(w(r_0)) \frac{d}{dr_0} S_B(r_0) \geq u'(w^{-1}(u(w(r)) - S_B(r_0))) - u'(w(r_0)) \) for all \( r_0 \in [0,1] \). Then the firm’s optimal cut-off \( r_0 \) is strictly lower when \( \phi < 1 \) than it is when \( \phi = 1 \).

**Proof.** For given \( \phi \) and \( r_0 \), total revenue is

\[
TR(r_0, S(r_0, \phi)) = p(r_0, \phi)q(r_0, \phi) = V(r_0, S(r_0, \phi))(\phi r_0),
\]

Recall the firm always chooses \( r_0 < 1 \) when \( \phi < 1 \), since \( V(1, S(1, \phi)) = 0 \) for all \( \phi < 1 \). Taking the derivative with respect to \( r_0 \) gives marginal revenue

\[
MR(r_0, S(r_0, \phi)) = \phi \left( V_1(r_0, S(r_0, \phi)) + V_2(r_0, S(r_0, \phi)) \frac{\partial}{\partial r_0} S(r_0, \phi) \right) r_0 + \phi V(r_0, S(r_0, \phi)). \tag{14}
\]

Proposition 1 implies \( S(r_0, \phi) < S_B(r_0) \) for all \( \phi < 1 \), so that \( 0 < V(r_0, S(r_0, \phi)) \leq V(r_0, S_B(r_0)) \). I now turn to the expression in large brackets in (14). From (3), write
\[ V(r, S(r_0, \phi)) = w(r) - u^{-1}(u(w(r)) - S(r_0, \phi)). \]

Differentiating with respect to the first argument and using \( dw/dr = -1/f(w) \) yields

\[ V_1(r, S(r_0, \phi)) = \frac{1}{f(w(r))} \left( -1 + \frac{u'(w(r))}{u'(u^{-1}(u(w(r)) - S(r_0, \phi)))} \right) > 0. \]

Differentiating with respect to the second argument yields

\[ V_2(r, S(r_0, \phi)) = \frac{1}{u'(u^{-1}(u(w(r)) - S(r_0, \phi)))} < 0, \]

so the expression in large brackets in (14) is equal to

\[ V_1(r_0, S(r_0, \phi)) + V_2(r_0, S(r_0, \phi)) \frac{\partial}{\partial r_0} S(r_0, \phi) = \frac{1}{f(w(r_0))} \left( -1 + \frac{f(w(r_0)) \frac{\partial}{\partial r_0} S(r_0, \phi) + u'(w(r))}{u'(u^{-1}(u(w(r)) - S(r_0, \phi)))} \right). \quad (15) \]

If \( u(c) \) is linear, then write \( u'(c) = \alpha \) for some constant \( \alpha > 0 \). When \( \phi = 1 \), (3) implies \( V(r_0, S_B(r_0)) = S_B(r_0)/\alpha \), and (15) simplifies to \( \frac{\partial S_B(r_0)}{\partial r_0} r_0/\alpha \). Hence \( MR(r_0, S_B(r_0)) = \frac{\partial}{\partial r_0}(rS_B(r_0))/\alpha \), which was shown to be strictly positive for all \( r_0 \) in the proof of Proposition 2. By continuity, it will continue to be strictly positive if \( \frac{\partial^i u(c)}{\partial c^i} < \epsilon \) for all \( i \geq 2 \), for \( \epsilon > 0 \) sufficiently small. Hence the firm chooses \( r_0 = 1 \) in the baseline.

If instead \( f(w(r_0)) \frac{\partial}{\partial r_0} S_B(r_0) \geq u'(u^{-1}(u(w(r)) - S(r_0, \phi))) - u'(w(r_0)) \) for all \( r_0 \in [0, 1] \), then (15) implies that again \( MR(r_0, S_B(r_0)) > 0 \) for all \( r_0 \). The firm again chooses \( r_0 = 1 \) in the baseline, which must be strictly larger than the optimal \( r_0 \) when \( \phi < 1 \).

If instead \( f(w(r_0)) \frac{\partial}{\partial r_0} S_B(r_0, \phi) < -u'(w(r_0)) \), then the numerator in (15) is negative in the baseline. The numerator is more negative at \( \phi < 1 \) than at \( \phi = 1 \), since \( \frac{\partial}{\partial r_0} S(r_0, \phi) < \frac{\partial}{\partial r_0} S_B(r_0) \). The denominator is positive and smaller at \( \phi < 1 \) than at \( \phi = 1 \), since \( S(r_0, \phi) < S_B(r_0) \). Taken together, this implies \( MR(r_0, S(r_0, \phi))/\phi < MR(r_0, S_B(r_0)) \), for all \( r_0 \in (0, 1] \). Hence for \( \phi < 1 \), the optimal \( r_0 \) must be strictly lower than in the baseline.

A firm that advertises more will generally also make the conspicuous good less exclusive, and thereby sell to a broader range of consumers. Total sales \( \phi r_0 \) will also increase. The result is consistent with Proposition 6, which showed that advertising tends to make consumption conformist rather than snobbish.

Recall that the firm charges a price \( p = V(r_0, S(r_0, \phi)) \) equal to the willingness to pay of the marginal consumer. When advertising increases the signaling value, Proposition 8 shows the firm moves to a poorer marginal consumer, even though \( V_{12} < 0 \) means willingness to pay increases more for the wealthy. This is
because advertising has the largest impact on the signaling value when the good is not exclusive, so when the marginal consumer is relatively poor.

Proposition 8 also suggests we are likely to observed limited advertising of goods that are highly exclusive. This is consistent, for example, with lower levels of advertising for super high-end watches x and y than for like x and y, and lower levels of advertising for Rolls Royce and Maserati cars than for BMW and Mercedes. An alternative explanation for the limited advertising of exclusive goods is that most consumers informed through ads simply wouldn’t buy. But this explanation is in fact complementary to one involving signaling, since it is precisely advertising’s low impact on sales that results in little change in willingness to pay.

To further explore the relationship between advertising cost, advertising levels and exclusivity, as well as that between advertising cost and price, I turn to simulations. I assume rank utility is quadratic, \( a(r) = a_0 + a_1 r + a_2 r^2 \), where \( a_0, a_1 \) and \( a_2 \) are constants, wealth \( w \) is uniformly distributed on an interval \([w, \bar{w}]\), advertising costs are \( K \beta \bar{\phi}^\beta \) for \( \beta > 1 \), and consumers exhibit constant relative risk aversion towards the numeraire good, \( u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha} \) for \( \alpha > 0 \), where \( u = \ln(c) \) for \( \alpha = 1 \). For given \( K \), I solve for the firm’s profit maximizing choice of \( \bar{\phi} \) and \( r_0 \), and I then let \( K \) vary. Doing so for a range of parameter values reveals three main points.

First, the firm’s optimal advertising level is often discontinuous in advertising cost, jumping directly from \( \bar{\phi} = 1 \) to \( \bar{\phi} < 1 \) when \( K \) exceeds a certain threshold. In this sense, a small increase in cost can lead to a significant drop in advertising levels. No discontinuity can arise in the baseline because revenues \( \bar{\phi} \bar{V}(r_0, S_B(r_0)) r_0 \) are linear in \( \bar{\phi} \), whereas costs \( K \bar{A}(\bar{\phi}) \) are convex. Now however, revenues are also convex in \( \bar{\phi} \) because advertising increases the signaling value. A discontinuity tends to occur when \( \beta \) is not too large, so when advertising costs themselves are not too convex.\(^5\) Figure 6 illustrates such a discontinuity when \( a_0 = 1, a_1 = 0, a_2 = -1, w = 0, \bar{w} = 10, \beta = 2 \) and \( \alpha = 1 \).

\( ^5 \)A necessary condition for a discontinuity at \( \bar{\phi} = 1 \) is that \( \lim_{\bar{\phi} \to 1} A'(\bar{\phi}) \) be finite, since then \( \bar{\phi} = 1 \) may be optimal for \( K \) sufficiently small. Note also that this discontinuity is unrelated to the discontinuity in signalling value from Proposition 4.

Figure 6
Second, the firm consistently makes the conspicuous good more exclusive as advertising costs rise and advertising levels fall, where the simulations all show the optimal \( r_0 \) decreasing monotonically in \( K \). This is the case regardless of whether consumption is conformist or snobbish, which suggests the result from Proposition 7 holds more generally. Moreover, exclusivity can also vary discontinuously with advertising cost, since any discrete drop from \( \phi = 1 \) to \( \phi < 1 \) is paired with a discrete drop in \( r_0 \). This is shown in Figure 7, for the same parameter values as above.

Third, the relationship between price and advertising cost is ambiguous, where the optimal \( p \) can either increase or decrease with \( K \). A rise in advertising costs leads the firm to decrease \( \phi \), which generates a change in \( r_0 \), holding \( p \) constant. Using \( r_0 = D(S(r_0, \phi), p) \) from (7), write
\[ dr_0 = \frac{\partial D}{\partial S} \frac{\partial S}{\partial r_0} dr_0 + \frac{\partial D}{\partial S} \frac{\partial S}{\partial p} dp, \]

so that

\[ \frac{dr_0}{d\phi} = \left( 1 - \frac{1}{\frac{\partial S}{\partial \phi} \frac{\partial S}{\partial r_0}} \right) \frac{\partial D}{\partial S} \frac{\partial S}{\partial \phi}. \]  

(16)

Since \( \frac{\partial D}{\partial S} > 0 \) and \( \frac{\partial S}{\partial \phi} > 0 \), comparing (13) and (16) shows that \( \frac{dr_0}{d\phi} \) and \( \frac{dr_0}{dp} \) have opposite signs. This means that holding price constant, a drop in advertising results in greater exclusivity whenever demand is downwards sloping. The drop in \( \phi \) and \( r_0 \) will also generate a new signaling value and a marginal consumer with higher wealth, both of which the firm takes into account when deciding how to respond to the increase in advertising cost: by how much to reduce \( \phi \), and in which direction to adjust \( p \). The simulations show that the optimal price can move in either direction. Figure 8 shows the case where price is decreasing in advertising cost, again for the same parameter values as above. Figure 9 show a different case where price is increasing in advertising cost.

Figure 8

Price as a Function of Advertising Cost, \( a(r) = 1 - r^2, a=1 \)

Figure 9
I conclude this section by considering the issue of welfare. The following result is similar to Proposition 4 from Corneo and Jeanne (1997), which considers prohibiting sale of the conspicuous good.

**Proposition 9.** For any \( r_0 > 0 \) and \( \phi > 0 \), the sum of individual utilities is strictly lower than when \( \phi = 0 \). The utility of every consumer is strictly lower than when \( \phi = 0 \) if

\[
 u(w(0)) - u(w(0) - V(r_0, S(r_0, \phi))) > \frac{\int_{r_0}^{r_0^*} a(r) \, dr}{r_0} - \int_0^1 a(r) \, dr.
\]

*Proof.* For given \( p, r_0 \) and \( \phi \), the sum of individual utilities is

\[
 \phi \int_0^{r_0} \left( u(w(r) - p) + s_0 \right) \, dr + (1 - \phi) \int_0^{r_0} \left( u(w(r)) + s_1 \right) \, dr + \int_{r_0}^1 \left( u(w(r)) + s_1 \right) \, dr.
\]

Substituting (4), (5) and \( p = V(r_0, S(r_0, \phi)) \) gives

\[
 \phi \int_0^{r_0} u(w(r) - V(r_0, S(r_0, \phi))) \, dr + (1 - \phi) \int_0^{r_0} u(w(r)) \, dr + \int_{r_0}^1 u(w(r)) \, dr + \int_0^1 a(r) \, dr. \tag{17}
\]

Since \( V(r_0, S(r_0, \phi)) > 0 \), (17) is strictly greater when \( r_0 > 0 \) and \( \phi > 0 \) than it is when \( \phi = 0 \).

A consumer who does not buy the conspicuous good when \( \phi > 0 \) is always better off when \( \phi = 0 \), since his status utility increases from \( s_1 \) to \( \int_0^1 a(r) \, dr \). A consumer with wealth \( w(r) \) who does buy is also better off if

\[
 u(w(r)) - u(w(r) - V(r_0, S(r_0, \phi))) \geq \frac{\int_{r_0}^{r_0^*} a(r) \, dr}{r_0} - \int_0^1 a(r) \, dr,
\]

where the left-hand side is increasing in \( r \) by \( u''(w) < 0 \). \( \square \)
Aggregate status utility is constant, where advertising and sale of the conspicuous good simply redistributes status from poor consumers who don’t buy to wealthy consumers who do. It also leads to lower consumption of the numeraire good, which reduces the sum of individual utilities. Wealthy consumers may be willing to pay such a high price to avoid low status that everyone end up worse off.

Corneo and Jeanne interpret the sum of individual utilities as social welfare if the conspicuous good is provided competitively, so if the firm earns zero profits. With this interpretation, advertising is excessive from a social perspective, as setting $\phi = 0$ is welfare improving. A possible critique is that advertising and sale of the conspicuous good might still increase each consumer’s utility if it is paired with appropriate zero-sum compensating transfers. However, advertising is certainly excessive when the condition stated in the proposition holds, as then setting $\phi = 0$ yields a Pareto improvement. This is in contrast to the standard result that a monopolist always underprovides informative advertising (Shapiro, 1980).

Corneo and Jeanne also consider the imposition of a per unit luxury tax on the conspicuous good. They assume price is fixed due to competitive pressures, and that a tax of $t > 0$ then brings the price to $p + t$. The tax is redistributed in a lump sum way to all consumers who buy. They show that a marginal increase in the tax will increase the utility of all consumers if and only if demand is locally downward sloping.

To address these issues in the current setting, I can interpret $K$ as being the advertising cost inclusive of a non-negative advertising tax. Keeping the price of the conspicuous good fixed, I show how increasing the advertising tax affects consumer welfare.

**Proposition 10.** Fix $p$ and $K$, and suppose the firm chooses the optimal $\phi$, with corresponding cut-off $r_0$. Interpret an increase in $K$ with an increase in advertising tax. Then if demand is locally downwards sloping, $\frac{dr_0}{dp} < 0$, a marginal increase of the tax will increase the sum of individual utilities. If demand is locally upwards sloping, $\frac{dr_0}{dp} > 0$, then it will increase the sum of individual utilities when $K$ is sufficiently high.

**Proof.** For a given $p$, $\phi$ and $r_0$, the sum of individual utilities is

$$\phi \int_0^{r_0} u(w_r - p) dr + (1 - \phi) \int_{r_0}^1 u(w_r) dr + \int_0^1 u(w_{r_0}) dr + \int_0^1 a(r) dr \quad (18)$$

By Proposition 7, a marginal increase in $K$ implies a marginal change in $\phi$. The rate of change of (18) with respect to $\phi$ is

$$\int_0^{r_0} u(w_r - p) - u(w_r) dr + \phi \left( u(w_{r_0} - p) - u(w_{r_0}) \right) \frac{dr_0}{d\phi}.$$

Both the integral and the difference in large brackets are negative, while comparing (13) and (16) shows that $\frac{dr_0}{d\phi}$ and $\frac{dr_0}{dp}$ have the opposite sign. Hence the entire expression is negative if $\frac{dr_0}{dp} < 0$, so if demand is
locally downwards sloping. It is also negative if \( \frac{\partial r_0}{\partial p} > 0 \), so if demand is locally upwards sloping, as long as \( \phi \) is sufficiently small. Again by Proposition 7, this is the case whenever \( K \) is sufficiently high.

The impact of an advertising tax is similar to a luxury tax when demand is downwards sloping. An increased advertising tax causes the firm to advertise less, to which consumers respond by reducing demand and making the conspicuous good more exclusive. Both \( r_0 \) and \( \phi \) fall, so total expenditure \( p\phi r_0 \) falls as well. Consumption of the numeraire good increases and consumers as a whole are better off. Unlike in Corneo and Jeanne, the tax does benefit all consumers, since wealthy consumers who no longer receive an ad are now unable to buy.

An advertising tax can be quite different from a luxury tax when demand is upwards sloping. Now \( r_0 \) rises but \( \phi \) still falls, so that total expenditure may still fall when \( \phi \) is low. With upward sloping demand, both types of taxes stimulate demand, but an advertising tax can still reduce sales by decreasing the number of informed consumers.

Consumers as a whole may also benefit from an advertising tax when the firm can optimally adjust both \( p \) and \( \phi \). For example, Figures 7, 8 and 9 show a case where \( p, \phi \) and \( r_0 \) would all fall with an increased tax, causing total expenditure \( p\phi r_0 \) to fall as well. A tax would also benefit every consumer if \( \phi = 1 \) and consumption was sufficiently conformist for demand to be everywhere upwards sloping.\(^6\)

4. Advertising and Recognition

Up until now, I have assumed that advertising is informative in the classic sense, transmitting information about price and product existence, and allowing consumers to buy. But a good deal of advertising also informs consumers by promoting product recognition. Advertising can help familiarize consumers with product appearance or brand image, allowing them to recognize and distinguish between different products, in particular when they see them displayed by others.

Recognition essentially means that a consumer can identify a good for what it is when he sees it. An uninformed consumer might be unable to distinguish a new, high-end smartphone from many other models. A consumer who recognizes the smartphone will not make that mistake and may be able to infer something about its owner. In this sense, broad recognition of a product is essential for consumers to be able to signal through their purchases.

\(^6\)The firm would then choose \( r_0 = 1 \) and sell to all consumers, and a tax leading to \( \phi < 1 \) would necessarily cause both \( p \) and \( r_0 \) to fall, since \( V(1, S(1, \phi)) = 0 \).
The importance of recognition is related to concerns in marketing over brand image: a brand can be thought of as an idea that is more powerful if widely shared. More people should therefore be familiar with the brand than just the consumers who buy (Kotler and Keller, 2008). For example, it is precisely because everyone knows BMW and what it stands for, even those who will never buy a BMW, that the brand has so much power (Kapferer, 2008). As described in the Introduction, this relationship between advertising, recognition and conspicuous consumption has so far received little formal attention.

To incorporate advertising’s role in promoting recognition into the analysis, I make the following change to the model. I now assume the signaling value is $S(r_0, \phi) = s_0(r_0) - s_1(r_0, \phi)$ multiplied by the fraction of consumers who receive an ad, so for now $\phi S(r_0, \phi)$. Recall the interpretation that $s_0$ and $s_1$ arise from the beliefs of a social contact with whom a consumer is matched after purchase. I now effectively assume the social contact can only observe the consumer’s purchase decision with a given positive probability, equal to the fraction of consumers who receive an ad. In fact, one can now think of the social contact as a randomly chosen consumer who can only recognize the good if he himself is informed. The ex-post signaling value is $s_0(r_0) - s_1(r_0, \phi)$ if this consumer received an ad and zero if he did not, giving expected signaling value $\phi S(r_0, \phi)$.

With this approach to recognition, advertising serves to ensure the conspicuous good is effectively visible. For the purchase of the conspicuous good to influence people’s beliefs, physical visibility is not enough. To be truly conspicuous, people must also recognize the good to understand what having it means, which creates another channel through which informative advertising can influence willingness to pay.

In many ways, advertising’s impact on the signaling value through increased recognition is simpler than its impact through increased sales. The latter was shown in Section 3 to depend on exclusivity, to be increasing at an increasing rate, to promote conformist over snobbish behavior, and so on. In contrast the relationship between recognition and signaling value is linear. For a given value of $S$, any change in advertising generates a proportional change in the signaling value.\(^7\)

Despite its relative simplicity, I now show that advertising’s impact on recognition leads to novel predictions when the firm can use targeted advertising. In practice, firms may be able to target costly ads at specific groups of consumers who are more likely to buy. Firms often do just that, putting great effort into selecting which of distinct audience to reach via specialized cable television, satellite radio, and magazines Esteban et al. (2006). Targeting is also becoming easier as technology improves (Johnson 2009, Esteves and Resende 2011).

I incorporate targeting into the analysis by allowing the firm to choose the subset of consumers to whom

\(^7\)Of course, the change in advertising also affects $S(r_0, \phi)$, both directly through $\phi$ and indirectly through $r_0$.\]
it advertises. The firm now chooses \( p, \phi \) and \( r_t \), where ads are randomly sent to a fraction \( \phi \) of consumers with rank on \([0, r_t] \), at cost \( Kr_tA(\phi) \). The firm therefore targets its ads on the \( r_t \) wealthiest consumers, where the value of \( r_t \) determines the degree of targeting. Exogenously setting \( r_t = 1 \) yields the original model.

I first show that when advertising does not promote recognition, the firm will target as much as possible.

**Proposition 11.** Suppose the firm can advertise randomly to consumers with rank \( r \in [0, r_t] \), and that informing a fraction \( \phi \) of these consumers costs \( KA(\phi, r_t) \). Suppose furthermore that the signaling value is \( S(r_0, \phi) \), given by (6). Then for any \( K > 0 \), the firm targets ads precisely on potential demand, \( r_t = r_0 \).

**Proof.** Consider any candidate equilibrium with \( p, \phi, r_t \) and corresponding \( r_0 \). Suppose first that \( r_t < r_0 \). Then a fraction \( \phi \) of consumers on \([0, r_t] \) buy, giving quantity sold \( \phi r_t \). By the definition of \( r_0 \), all consumers \( r < r_0 \) are willing to pay strictly more than \( p \), so the firm can marginally increase its price with no effect on quantity sold. Hence \( r_t < r_0 \) cannot be optimal.

Suppose instead that \( r_t > r_0 \). A mass \( \phi r_t \) of consumers receive an ad, out of which a fraction \( r_0/r_t \) demand the good. Just as with \( r_t = 1 \), a fraction \( \phi \) of consumers on \([0, r_0] \) buy, with quantity sold \( \phi r_0 \), signaling value \( S(r_0, \phi) \) and price \( p = V(r_0, S(r_0, \phi)) \). Total advertising cost is \( Kr_tA(\phi) \).

If the firm keeps \( p \) and \( \phi \) constant but sets \( r_t = r_0 \), then a fraction \( \phi \) of consumers on \([0, r_0] \) still receive an ad. Moreover, from (6), the same signaling value \( S(r_0, \phi) \) is still consistent with the same cut-off \( r_0 \). Hence quantity sold remains \( \phi r_0 \), but total advertising costs decrease: \( Kr_tA(\phi) < Kr_0A(\phi) \). \( \square \)

If advertising does not promote recognition, the firm will use targeted advertising because it is the cheapest way to inform potential demand. Targeting ads to the \( r_0 \) wealthiest consumers ensures that everyone who receives an ad is also willing to buy. Any larger choice of \( r_t \) would mean ads are wasted, in the sense that some consumers who receive them would not buy. These wasted ads would have no direct impact on sales and no indirect impact on the signaling value through the mechanism analyzed in Section 3. Proposition 11 is consistent with results from Hernandez-Garcia (1997), Esteban et al. (2001) and Esteban et al. (2006), who show that a monopolist uses targeted advertising when it is the cheapest way to inform potential demand.

The situation can be quite different when advertising promotes recognition, so that the signaling value is proportional to the fraction of informed consumers.

**Proposition 12.** Consider the same advertising technology as in Proposition 11, but suppose the signaling value is now \( \phi r_t S(r_0, \phi) \). Then in the limit as \( K \) tends to zero, the firm advertises as broadly as possible, \( r_t = 1 \).
Proof. For any \( r_0 \), the signaling value \( \phi r_t S(r_0, \phi) \) is strictly increasing in \( \phi \) and \( r_t \). The firm can charge price \( p = V(r_0, \phi r_t S(r_0, \phi)) \), to obtain quantity sold \( \phi r_0 \). Profits are then

\[
V(r_0, \phi r_t S(r_0, \phi)) \phi r_0 - KA(\phi, r_t).
\]

By \( V_2 > 0 \), a marginal increase in \( r_t \) or \( \phi \) will strictly increase revenue by an amount that is independent of \( K \). It will also increase advertising cost by an amount that is proportional to \( K \). Hence in the limit as \( K \) tends to zero, the optimal \( r_0 \) and \( \phi \) tend to 1. \( \square \)

When advertising promotes recognition and costs are low, the firm no longer wants to target ads precisely on the potential demand, but instead to advertise as broadly as possible. The reason is that ads received by consumers who don’t buy are no longer wasted. They allow these consumers to recognize the conspicuous good, so that others can signal their wealth through their purchases.

The result suggests that firms may still have reasons to advertise broadly, even though targeting technology is available, if broad recognition is important for strengthening brand image. This idea is also expressed in Miller (2009) in the context of luxury goods:

The luxury brands with the highest brand equity ... advertise in Vogue and GQ not so much to inform rich potential consumers that they exist, but to reassure rich potential consumers that poorer Vogue and GQ readers will recognize and respect these brands when they see them displayed by others. (Miller 126)

Kapferer and Bastien (2009) make a similar point, espousing what they call an “anti-law” of marketing for luxury brands: it is important that more people be familiar with a brand than those who are likely to buy, so traditional advertising campaigns may be ineffective if they focus only on the target market.

5. Duopoly

I now consider how informative advertising impacts willingness to pay under imperfect competition. I look at a setting with horizontal product differentiation, with two firms located at either end of a Hotelling line. A unit mass of consumers, each with wealth \( w_H \), is uniformly distributed over this line. A consumer’s position on the line is denoted by \( x \in [0, 1] \), where transport costs are \( tx \) to buy from Firm 1 and \( t(1-x) \) to buy from Firm 2. A further mass \( m > 0 \) of consumers have wealth \( w_L = 0 \) and so cannot buy from either firm.

29
The status utility from being identified as having high wealth is \( a(w_H) \equiv a > 0 \) and as having low wealth is \( a(w_L) \), where I normalize the latter to zero.\(^8\)

Firms 1 and 2 simultaneously chooses price and advertising, \((p_1, \phi_1)\) and \((p_2, \phi_2)\). Consumers with high wealth then choose whether to buy a single unit of a good from one firm from which they receive an ad.

Consumers who buy either good all have wealth \( w_H \), so the status utility from buying either good is \( a \).

If total quantity sold is \( Q \), then a mass \( 1 - Q + m \) consumers buy nothing, of which \( m \) have wealth \( w_L \) and \( 1 - Q \) have wealth \( w_H \). The status utility from buying nothing is therefore \((1 - Q)a/(1 + m - Q)\). The signaling value is the difference between the two,

\[
S = \frac{am}{1 + m - Q} > 0. \tag{19}
\]

The signaling value is the same for both goods and increasing in total quantity sold, from \( S(0) = am/(1 + m) \) to \( S(1) = a \). A consumer with high wealth located at position \( x \) on the Hotelling line is willing to pay \( V(S) - tx \) for the good from Firm 1, and \( V(S) - t(1 - x) \) for the good from Firm 2. I assume throughout that \( V(a) < w_H \). This means I can abstract away from the consumers’ budget constraint and just assume \( V \) is an increasing function of \( S \).

Looking at expression (19) for the signaling value, I can say the following. First, advertising may increase willingness to pay under duopoly, but only to the extent that it generates higher total sales. If a firm’s advertising simply steals sales away from its rival then willingness to pay will remain unchanged. Second, advertising that does increase willingness to pay cannot generate a competitive advantage, since willingness to pay for the rival’s good will increase by the same amount.

Let \( r_1 \in [0, 1] \) denote the largest value of \( x \) for which the consumer located at this point on the Hotelling line weakly prefers buying from Firm 1 to buying nothing. Similarly, let \( r_2 \in [0, 1] \) denote the smallest value of \( x \) for which the consumer located at this point weakly prefers buying from Firm 2 to buying nothing. Defined in this way, \( r_1 \) and \( r_2 \) are the marginal consumers for Firm 1 and Firm 2. The values of \( r_1 \) and \( r_2 \), and therefore of quantity sold \( q_1 \) and \( q_2 \), will depend on \((p_1, \phi_1)\) and \((p_2, \phi_2)\).

For any given prices and advertising, quantity sold is determined by the equilibrium behavior of consumers. This quantity sold may not be unique. Consumption externalities mean that consumers face a coordination game, where the signaling value of both goods is increasing in the number of consumers who

\(^8\)The assumption that wealth is either high or low, and that low wealth consumers never buy, is in line with Kuksov and Xie (2010). The resulting analysis then also resembles Hamilton (2009), who considers informative advertising with horizontal differentiation but without social status. The difference is that here consumer valuations are not constant, but instead depend on prices and advertising.

30
buy either good. This means willingness to pay, and hence quantity sold $Q$, may depend on consumers’ expectations about quantity sold $Q^e$. Consumer equilibrium behavior implies expectations are correct, $Q^e = Q(Q^e)$.

The game played by consumers play is supermodular, that is with strategic complementarities. By well known results for such games (see, e.g., Vives (1999)), this means that for any prices and advertising, there must exist at least one pair $(r_1, r_2)$ consistent with consumer equilibrium behavior and that is stable in the following sense: if expected quantity sold differs from the equilibrium level by $\epsilon > 0$ sufficiently small, then the resulting quantity sold will differ from the equilibrium level by strictly less than $\epsilon$. If $r_i \in (0, 1)$ for $i \in \{1, 2\}$, this amounts to $Q(Q^e)$ crossing the 45 degree line from above. The results that follow will apply to such stable equilibria.

I consider symmetric equilibria, where $p_1 = p_2$, $\phi_1 = \phi_2$ and $r_1 = r_2$. Following Hamilton (2009), I distinguish between three types of equilibria: local monopoly if $r < 1/2$, incomplete coverage if $1/2 < r < 1$, and complete coverage if $r = 1$. In local monopoly, a firm does not face competition over its marginal consumer, who prefers buying nothing to buying from its rival. In incomplete coverage, firms compete over all fully informed consumers near the center of the Hotelling line, but not over consumers at either extreme. In complete coverage, firms compete over all fully informed consumers.

The following results look at how a firm’s demand per informed consumer depends on its own advertising and the advertising of its rival. Specifically, they consider the rate of change of $q_i/\phi_i$ with respect to $\phi_i$ and $\phi_j$, evaluated at a particular equilibrium. In the standard setting with informative advertising but without status effects, so where consumer valuations are constant, demand per informed consumer would be independent of a firm’s own advertising and weakly decreasing in the rival firm’s advertising. In contrast, the results here will depend on whether the equilibrium is characterized by local monopoly, incomplete coverage or complete coverage.

**Proposition 13.** Suppose $V'(a) < t/2$. Then any pure strategy equilibrium involves local monopoly, $r < 1/2$. For each firm $i$, demand per informed consumer, $q_i/\phi_i$, evaluated in equilibrium, is strictly increasing in both $\phi_i$ and $\phi_j$.

A sufficient condition for existence is that for any given pair $p_1 = p_2 = p$ and $\phi_1 = \phi_2 = \phi$, the resulting value of $r_1 = r_2 = r$ is unique. This will be the case if $V' \frac{\partial S}{\partial Q} < t/2$, for all $Q \in (0, 1]$.

**Proof.** Local monopoly means $r_1 < 1/2$ and $r_2 < 1/2$, so that $q_1 = \phi_1 r_1$ and $q_2 = \phi_2 r_2$. Total demand is $Q = \phi_1 r_1 + \phi_2 r_2$.

Firm $i$ profits are $\pi_i = p_i q_i - A(\phi_i)$, for $i \in \{1, 2\}$. The relationship between $p_1$, $\phi_1$, $p_2$, and $\phi_2$, on the one hand, and $r_1$ and $r_2$, on the other is
for \( i \in \{1, 2\} \), with \( S \) given by (19). Each firm can charge a price equal to \( V \) evaluated at the signaling value minus the transport cost of the marginal consumer.

By \( V(S) > 0 \), \( A(0) = 0 \) and \( A'(0) = 0 \), Firm \( i \) can always earn strictly positive profits by choosing \( p_i \) and \( \phi_i \) sufficiently small. Hence each firm must earn strictly positive profits in equilibrium. We can therefore restrict attention to \( \phi_i \in (0, 1) \) and to prices \( p_i \) such that \( r_i \in (0, V(a)/t) \). By \( V(a) < t/2 \), this implies \( r_i < 1/2 \) as required in local monopoly.

Rearranging (20) gives

\[
r_i = \frac{1}{t} \left( V(S) - p_i \right),
\]

and \( Q = \phi_1 r_1 + \phi_2 r_2 \) implies

\[
Q = \left( \frac{\phi_1 + \phi_2}{t} \right) V(S(Q^*)) - \left( \frac{p_1 + p_2}{t} \right).
\]

Here I explicitly distinguish between realized quantity sold \( Q \) and expected quantity sold \( Q^* \). The condition for consumer equilibrium behavior is \( Q_e = Q \). For stability, the derivative of the right-hand side of (22) with respect to \( Q_e \) must be strictly less than one when it crosses the 45 degree line, so \( V' \frac{dS}{dQ} < t/(\phi_1 + \phi_2) \).

The condition in a symmetric equilibrium is \( V' \frac{dS}{dQ} < t/2\phi \).

Now consider how \( r_1 \) and \( r_2 \) vary with a marginal change in \( \phi_1 \) or \( p_1 \). By (21), write

\[
trdr_1 = V' \frac{dS}{dr_1} dr_1 + V' \frac{dS}{dr_2} dr_2 + V' \frac{dS}{d\phi_1} d\phi_1 - dp_1,
\]

where \( tdr_2 = tdr_1 + dp_1 \). Rewriting in matrix form gives

\[
\begin{bmatrix}
  t - V' \frac{dS}{dr_1} & -V' \frac{dS}{dr_2} \\
  -V' \frac{dS}{dr_1} & t - V' \frac{dS}{dr_2}
\end{bmatrix}
\begin{bmatrix}
  dr_1 \\
  dr_2
\end{bmatrix}
= \begin{bmatrix}
  V' \frac{dS}{d\phi_1} d\phi_1 - dp_1 \\
  V' \frac{dS}{d\phi_1} d\phi_1
\end{bmatrix},
\]

the solution to which is

\[
\begin{bmatrix}
  dr_1 \\
  dr_2
\end{bmatrix}
= \frac{1}{\Delta}
\begin{bmatrix}
  t - V' \frac{dS}{dr_1} & V' \frac{dS}{dr_2} \\
  V' \frac{dS}{dr_1} & t - V' \frac{dS}{dr_2}
\end{bmatrix}
\begin{bmatrix}
  V' \frac{dS}{d\phi_1} d\phi_1 - dp_1 \\
  V' \frac{dS}{d\phi_1} d\phi_1
\end{bmatrix},
\]

where \( \Delta = (t - V' \frac{dS}{dr_1})(t - V' \frac{dS}{dr_2}) - (V' \frac{dS}{dr_1})(V' \frac{dS}{dr_2}) \). By \( Q = \phi_1 r_1 + \phi_2 r_2 \), we have \( \Delta = (t - V' \frac{dS}{dQ} \phi_1)(t - V' \frac{dS}{dQ} \phi_2) - (V' \frac{dS}{dQ} \phi_1)(V' \frac{dS}{dQ} \phi_2) \). In a symmetric equilibrium, \( \Delta = t(t - 2V' \frac{dS}{dQ} \phi) \) which by stability is
strictly positive. Moreover, \( t - V' \frac{dS}{dQ} \), \( V' \), \( \frac{dS}{dQ} \), and \( \frac{dS}{dQ} \) are all strictly positive. This implies \( \frac{\partial r_1}{\partial \phi_1} > 0 \), \( \frac{\partial r_2}{\partial \phi_1} > 0 \), \( \frac{\partial r_1}{\partial p_1} < 0 \) and \( \frac{\partial r_2}{\partial p_1} < 0 \).

Firm \( i \) profits are \( \pi_i = p_i \phi_i r_i - A(\phi_i) \), yielding first order conditions

\[
\frac{\partial \pi_i}{\partial p_i} = r_i + p_i \frac{\partial r_i}{\partial p_i} = 0, \tag{23}
\]

and

\[
\frac{\partial \pi_i}{\partial \phi_i} = p_i r_i + p_i \phi_i \frac{\partial r_i}{\partial \phi_i} - A'(\phi_i) = 0, \tag{24}
\]

for \( i \in \{1, 2\} \). In a symmetric equilibrium, \( p_1 = p_2 \) and \( \phi_1 = \phi_2 \), giving two equations in two unknowns. Expression (23) implicitly defines a function \( p = g(\phi) \), and expression (24) implicitly defines a function \( \phi = h(p) \). An equilibrium pair \((p^*, \phi^*)\) satisfies \( p^* = g(\phi^*) \) and \( \phi^* = h(p^*) \).

If \( p > V(a) \), then (21) implies \( r = \frac{\partial r}{\partial \phi} = 0 \), so that \( h(p) = 0 \). If \( p = 0 \), then again \( h(p) = 0 \). Moreover, if \( p = V(am/(m+1))/2 \), then (21) implies \( r > 0 \), so that \( h(p) > 0 \).

If \( \phi = 0 \), then (19) and (21) imply \( r = \frac{1}{t} (V(\frac{am}{m+1}) - p) \), so that \( g(\phi) = V(am/(m+1))/2 \). Hence \( p = g(\phi) \) and \( \phi = h(p) \) must intersect at some \( p^* \) and \( \phi^* \), as long as both \( f \) and \( g \) are continuous. This will be the case if \( r \), \( \frac{\partial r}{\partial p} \) and \( \frac{\partial r}{\partial \phi} \) are continuous functions of \( p \) and \( \phi \). A sufficient condition is that, for every pair \( p_1 = p_2 = p \) and \( \phi_1 = \phi_2 = \phi \), the resulting \( r \) is unique. This \( r \) must be stable as the unique equilibrium of a supermodular game, and so from the above will vary smoothly with \( p \) and \( \phi \).

In a symmetric equilibrium, (22) implies

\[
Q = \frac{2\phi}{t} \left( V(S(Q)) - 2p \right).
\]

If \( V' \frac{dS}{dQ} < t/2 \), then the derivative of the right-hand side is always strictly less than one. Hence the right-hand side crosses the 45 degree line at most once on \( Q \in (0, 1] \). This implies that \( Q \) and \( r = Q/2\phi \) are unique.

To understand this result and those that follow, it is helpful to divide total quantity sold into three groups: captive consumers for Firm 1, captive consumers for Firm 2, and consumers over whom the firms compete. Captive consumers buy from the firm in question and their best outside option is to buy nothing, either because they don’t receive an ad from the firm’s rival or because they lie too far away from the rival on the Hotelling line. Consumers over whom firms compete receive ads from both firms, and they prefer buying from either firm over buying nothing.
An increase in a firm’s advertising will inform some consumers who previously didn’t buy at all. Those who lie close enough on the Hotelling line become captive and buy, which increases the firm’s quantity sold while leaving quantity sold per informed consumer unchanged. Total quantity sold also increases, which reduces the social status from buying nothing, and by (19) increases the signaling value of both goods. This has no impact on the consumers over whom firms compete, but it can create new captive consumers for each firm, as some who previously bought nothing now find it sufficiently attractive to buy. Overall, this demand effect increases demand per informed consumer for both firms.

An increase in advertising also informs some of the rival’s captive consumers. If firms can compete over these newly informed consumers, then some will switch away from the rival and buy from the firm instead. This again increases the firm’s quantity sold while leaving quantity sold per informed consumer unchanged. Here it has no impact on the signaling value of either good, since it simply shifts sales away from the rival. Overall, this business stealing effect leaves the firm’s demand per informed consumer unchanged, and causes the rival’s demand to drop.

With local monopoly, \( r < 1/2 \), the business stealing effect vanishes because there are no consumers over whom firms compete. However, for each firm, there are consumers marginally further away than \( r \) on the Hotelling line who were previously just unwilling to buy. A marginal increase in a firm’s advertising generates sales from these consumers by increasing the signaling value, so the demand effect is non-zero. The net effect is therefore to increase demand per informed consumer for both firms.

An consequence of Proposition 9 is that each firm’s profits are strictly higher than under monopoly, in the equilibria with the highest and lowest quantity sold. The reasoning is as follows. If Firm 1 adjusts its price and advertising from their equilibrium levels to their optimal levels under monopoly, and that these differ from one another, then Firm 1 profits will strictly decrease. If they don’t differ from one another, then a marginal increase in the price of Firm 2 will still cause Firm 1 profits to strictly decrease. Either way, a further increase of \( p_2 \) large enough for Firm 2 demand to equal zero yields precisely monopoly profits for Firm 1. But \( p_2 \) is the cost of consumers buying from Firm 2 in a supermodular game. By well known comparative static results in such games, an increase in \( p_2 \) must weakly decrease demand for both firms in the equilibrium with highest and lowest quantity sold. This implies monopoly profits for Firm 1 are strictly lower than equilibrium profits.

The following result shows that the impact of advertising is quite different under complete coverage.

**Proposition 14.** Suppose \( V(\alpha/(m+1)) \) is sufficiently large. Then a unique equilibrium exists, and it involves complete coverage, \( r = 1 \). For each firm \( i \), demand per informed consumer, \( q_i/\phi_i \), evaluated in equilibrium, is independent of \( \phi_i \) and is strictly decreasing in \( \phi_j \).
Proof. Complete coverage means $r = 1$, so that

$$1 < \frac{1}{t}(V(S) - p_i).$$  \hspace{1cm} (25)

Demand for Firm $i$ is

$$q_i = \phi_i(1 - \phi_j) + \phi_i\phi_j\left(\frac{t + p_j - p_i}{2t}\right),$$  \hspace{1cm} (26)

for $i \in \{1, 2\}$. It consists of the $\phi_i(1 - \phi_j)$ consumers who only receive an ad from Firm $i$, and the fraction of the $\phi_i\phi_j$ consumers who are fully informed who lie sufficiently close to Firm $i$ on the Hotelling line.

Demand for both firms is independent of the precise signaling value $S$, so equilibrium values of $p$ and $\phi$ do not depend on $V(S)$. Hence by $S \geq am/(m + 1)$, (25) will hold whenever $V(am/(m + 1))$ is sufficiently large. The equilibrium outcome is identical to the case where $V$ is a sufficiently large constant, so existence and uniqueness follow directly from Hamilton (2009).

By (26), it is immediate that $\frac{\partial(q_i/\phi_i)}{\partial \phi_i} = 0$ and $\frac{\partial(q_i/\phi_i)}{\partial \phi_j} = -1/2 < 0$ in a symmetric equilibrium.

With complete coverage, $r = 1$, firms compete over all fully informed consumers, so the business stealing effect is non-zero. An increase in a firm’s advertising will increase both total quantity sold and the signaling value, but the marginal consumer was already willing to buy in equilibrium. The increase in signaling value doesn’t generate any increase in sales, so the demand effect vanishes. The net effect is therefore to leave the firm’s demand per informed consumer unchanged, and to decrease demand for its rival.

The final result considers the case of incomplete coverage.

**Proposition 15.** In any equilibrium with incomplete coverage, $r \in (1/2, 1)$, demand per informed consumer, $q_i/\phi_i$, evaluated in equilibrium, is increasing in $\phi_i$. It is increasing in $\phi_j$ if $r$ is sufficiently close to $1/2$, and is decreasing in $\phi_j$ if $\phi_j$ is sufficiently close to 1.

Proof. Incomplete coverage means $1/2 < r < 1$. Demand for Firm $i$ is

$$q_i = \phi_i(1 - \phi_j)r_1 + \phi_i\phi_j\left(\frac{t + p_j - p_i}{2t}\right),$$  \hspace{1cm} (27)

for $i \in \{1, 2\}$, giving total demand

$$Q = \phi_1(1 - \phi_2)r_1 + \phi_1\phi_2(1 - \phi_1)r_2 + \phi_1\phi_2\left(\frac{t + p_j - p_i}{2t}\right).$$  \hspace{1cm} (28)

Restating (21),
for \( i \in \{1, 2\} \), where \( S = \frac{am}{1 + m - Q} \). Substituting (21) into (28) gives

\[
Q = \phi_1(1 - \phi_2)\frac{1}{t}(V(S(Q)) - p_1) + \phi_2(1 - \phi_1)\frac{1}{t}(V(S(Q)) - p_2) + \phi_1\phi_2\left(\frac{t + p_i - p_i}{2t}\right). \tag{29}
\]

The condition for stability is that the derivative of the right-hand side of (29) with respect to \( Q \) is less than 1. In a symmetric equilibrium, this amounts to \( V' \frac{dS}{dQ} < \frac{t}{2\phi(1-\phi)} \).

Consider (21), and suppose there is a marginal change in \( \phi_1 \). By the exact same reasoning as in the proof of Proposition 9, we have

\[
\begin{bmatrix}
\Delta_1 \\
\Delta_2
\end{bmatrix}
= \frac{1}{\Delta}
\begin{bmatrix}
V'(S_1) & V'(S_2)
\end{bmatrix}
\begin{bmatrix}
\frac{dS}{d\phi_1} \\
\frac{dS}{d\phi_1}
\end{bmatrix},
\]

where \( \Delta = (t - V'(S_1))(t - V'(S_2)) - (V'(S_2))(V'(S_1)). \) By (28), write \( \Delta = t(t - 2V'(S_2)\phi_1(1-\phi)) \) which by stability is strictly positive. All entries in the matrix are positive, as is \( V' \frac{dV}{d\phi} \), so that \( \partial r_1/\partial \phi_1 > 0 \) and \( \partial r_2/\partial \phi_1 < 0 \).

When \( p_i = p_j \), (27) implies

\[
q_1 = \phi_1(1 - \phi_2)r_1 + \frac{\phi_1\phi_2}{2},
\]

and

\[
q_2 = \phi_2(1 - \phi_1)r_2 + \frac{\phi_2\phi_1}{2}.
\]

This yields

\[
\frac{\partial(q_1/\phi_1)}{\partial \phi_1} = (1 - \phi_2)\frac{\partial r_1}{\partial \phi_1},
\]

which is strictly positive by \( \partial r_1/\partial \phi_1 > 0 \). It also yields

\[
\frac{\partial(q_2/\phi_2)}{\partial \phi_1} = (1 - \phi_1)\frac{\partial r_2}{\partial \phi_1} - (r_2 - \frac{1}{2}),
\]

where both terms on the right-hand side are strictly positive, by \( r_2 > 1/2 \) and \( \partial r_2/\partial \phi_1 > 0 \). When \( r \) approaches 1/2, the second term tends to zero while the first term does not, so that \( \partial(q_2/\phi_2)/\partial \phi_1 > 0 \). When \( \phi_1 \) approaches 1, the first term tends to zero while the second term does not, so that \( \partial(q_2/\phi_1)/\partial \phi_1 < 0 \).
With incomplete coverage, $1/2 < r < 1$, both the demand effect and the business are non-zero. This means that a marginal increase in a firm’s advertising will always increase its own demand per informed consumer, but it will have an unambiguous impact on the demand of its rival.

Whether this impact is positive or negative will depend on which effect dominates. The business stealing effect becomes small when $r$ is close to $1/2$. There are then very few consumers over whom the firms can compete, so a firm’s advertising will increase its rival’s demand. In contrast, the demand effect becomes small when $\phi$ approaches 1. High levels of advertising means there are few captive consumers, so increased advertising will decrease its rival’s demand.

6. Conclusion

This paper examines the role of informative advertising when brand image is endogenously determined by the purchases of status-conscious consumers. It helps to bridge the gap between the informative and persuasive views of advertising, by showing that purely informative advertising can increase willingness to pay.

A monopolist produces a conspicuous good that consumers can purchase to signal their wealth. In equilibrium, consumers are motivated to buy for two reasons: buying suggests they are wealthy, and not buying would suggest they are poor. Advertising informs consumers and allows a greater number of those who are wealthy to buy. This in turn increases the signaling incentive to buy the conspicuous good, since consumers who don’t are more closely identified as poor.

In this way, informative advertising increases willingness to pay by increasing the stigma associated with not buying. The potential impact of advertising is highest when the good is not exclusive, it can be discontinuous in the advertising level, and increased advertising tends to promote conformist over snobbish behavior.

Appendix

Proof of Proposition 8. Profits are

$$\pi = \phi V(r_0, S(r_0, \phi))r_0 - KA(\phi),$$

giving marginal revenue
\[ \frac{\partial}{\partial \phi} \pi = V(r_0, S(r_0, \phi))r_0 + \phi V_2(r_0, S(r_0, \phi)) \frac{\partial}{\partial r_0} S(r_0, \phi) - KA'(\phi). \]

Marginal revenue is strictly positive at \( \phi = 0 \) for any \( r_0 \), since \( V(r_0, S(r_0, 0)) > 0 \), \( A(0) = 0 \) and \( A'(0) \). For any \( K \), the firm finds it optimal to choose some \( \phi > 0 \) and \( r_0 > 0 \) and earn positive profits, rather than to choose \( \phi = 0 \) or \( r_0 = 0 \) and earn zero profits. Moreover, \( KA'(\phi) \) is increasing without bound in \( K \). For any given \( \phi > 0 \) and \( r_0 > 0 \), marginal revenue will be negative when \( K \) is sufficiently large. This implies \( \lim_{K \to \infty} \phi(K) = 0 \).

Now suppose that for given \( K \), both first order conditions hold at the optimal \( \phi \) and \( r_0 \), so \( \frac{\partial}{\partial r_0} \pi = 0 \) and \( \frac{\partial}{\partial \phi} \pi = 0 \). Taking the differential of each first order condition gives

\[
\frac{dr_0}{dK} \left[ \frac{\partial^2}{\partial r_0^2} \pi \right] + \frac{d\phi}{dK} \left[ \frac{\partial^2}{\partial r_0 \partial \phi} \pi \right] = 0,
\]

and

\[
\frac{dr_0}{dK} \left[ \frac{\partial^2}{\partial r_0 \partial \phi} \pi \right] + \frac{d\phi}{dK} \left[ \frac{\partial^2}{\partial \phi^2} \pi \right] = A'(\phi).
\]

Solving yields

\[
\frac{dr_0}{dK} = \left( \frac{\partial^2}{\partial r_0^2} \pi - \frac{1}{\partial^2 \phi \partial r_0^2 \pi} \right) \frac{\partial^2}{\partial r_0 \partial \phi} \pi A'(\phi),
\]

and

\[
\frac{d\phi}{dK} = \left( \frac{\partial^2}{\partial r_0^2} \pi - \frac{1}{\partial^2 \phi \partial r_0^2 \pi} \right) \frac{\partial^2}{\partial \phi^2} \pi A'(\phi).
\]

The second order condition implies \( \frac{\partial^2}{\partial r_0^2} \pi < 0 \) and \( \frac{\partial^2}{\partial r_0^2} \pi \frac{\partial^2}{\partial \phi^2} \pi - \frac{1}{\partial^2 \phi \partial r_0^2 \pi} > 0 \), so that \( \frac{d\phi}{dK} < 0 \).

Suppose instead that \( \frac{\partial}{\partial \phi} \pi > 0 \) at the optimal \( \phi \) and \( r_0 \). In this case, \( \phi = 1 \). Since \( \frac{\partial}{\partial \phi} \pi \) is continuous in \( K \) and \( r_0 \), \( \frac{\partial}{\partial \phi} \pi > 0 \) still holds after a marginal change in \( K \) and \( r_0 \). Hence \( \phi = 1 \) remains optimal.

Finally, suppose \( \frac{\partial}{\partial \phi} \pi = 0 \) but \( \frac{\partial}{\partial r_0} \pi > 0 \). This implies \( r_0 = 1 \). Since \( \frac{\partial}{\partial r_0} \pi \) is continuous in \( K \) and \( \phi \), \( \frac{\partial}{\partial r_0} \pi > 0 \) still holds after a marginal change in \( K \) and \( \phi \). Hence \( r_0 = 1 \) remains optimal. Plugging in \( \frac{dr_0}{dK} = 0 \) above yields \( \frac{d\phi}{dK} = A'(\phi)/(\partial^2 \phi \partial r_0^2 \pi) \), which is negative by the second order condition for \( \phi \).

**References**


