

Which Inequality?

The Inequality of Endowments Versus the Inequality of Rewards*

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March, 2008

Abstract

Society often allocates valuable resources - such as prestigious positions, salaries, or marriage partners - via tournament-like institutions. In such situations, inequality affects incentives to compete and hence has a direct effect on equilibrium choices and hence material outcomes. We introduce a new distinction between inequality in initial endowments (e.g. ability, inherited wealth) and inequality of what one can obtain as rewards (e.g. prestigious positions, money). We show that these two types of inequality have opposing effects on equilibrium behavior and wellbeing. Greater inequality of rewards tends to hurt most people – both the middle class and the poor, – who are forced into greater effort. In contrast, greater inequality of endowments tends to benefit the middle class. Thus, which type of inequality is considered hugely affects the correctness of our intuitions about the implications of inequality.

Keywords: inequality, endowments, rewards, relative position, ordinal rank, games, tournaments, dispersive order, star order.

JEL codes: C72, D63, D62, D31.

*We thank Helmut Bester, Simon Clark, Kai Konrad, Benny Moldovanu, Andrew Oswald, Prasanta Pattanaik, Mike Peters, József Sákovics and participants at the Edinburgh social economics workshop and the Public Economic Theory conference, Marseille for helpful discussions. Ed Hopkins thanks the Economic and Social Research Council, Research Fellowship Scheme award reference RES-000-27-0065, and a Leverhulme Trust Study Abroad Fellowship for support.

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1 Introduction

Perhaps there is no other economic debate older than that over inequality. As Sen (1980) points out, while most have agreed that some form of equality is desirable, there has been less consensus on what should be equalized. There is even debate over what is meant by equality and inequality (see Sen (1980), Phelps Brown (1988), Roemer (1996), Lamont (2003) and many others). Despite this diversity of opinion, typically equality is treated as a moral question. There may be some distributions of income or of wealth or some methods of distribution of endowments which are simply unfair. In contrast, in economics, the second fundamental welfare theorem seemed to separate these moral issues from the mainstream of economic analysis, the study of efficiency. Only recently it has been suggested that people have “social preferences”, so that they care directly about what others receive as well as their own income or consumption. Differing formulations have been proposed by Frank (1985), Fehr and Schmidt (1999) and Charness and Rabin (2002) amongst many.

Here, we take a purely economic approach and examine a model where individuals care only about their own consumption, yet inequality has a material effect on market outcomes. We assume a society where individuals differ in terms of initial endowments, whether it is innate ability, education received or inherited wealth. Second, the rewards that individuals receive as a result of their achievements also vary. A fixed set of rewards, that could represent cash prizes, places at a prestigious university, attractive jobs, desirable spouses, social esteem, monopoly rents or any combination of these, is assigned by a tournament. Individuals, whose endowments are private information, make a simultaneous decision over effort or performance. Then each individual is given a reward according to his rank in the distribution of performance: first prize is given to first place, second prize to second place, and so on.

Such a tournament creates important positional externalities, to obtain a top reward one must occupy a top position, and by doing so one excludes others from that position and hence that reward. This induces competitors to behave as though they had a desire for high rank or status, an observation first due to Cole, Mailath and Postlewaite (1992), discussed further by Postlewaite (1998). In turn, this leads to equilibrium effort being inefficiently high and equilibrium utility being inefficiently low. Crucially, these externalities also imply that the equilibrium choice of effort and equilibrium utility depend on both the initial distribution of endowments and the distribution of rewards. Therefore, there is no need to appeal to any notion of justice for equality to matter. It matters because what others have affects the job one gets, the wage one is paid and the amount of leisure one takes.

Thus, both the distribution of endowments and the distribution of rewards affect individual choices and equilibrium utility. However, we find that changes in the inequality of endowments have the opposite effect to changes in the inequality of rewards. An increase in the equality of competitors’ endowments raises the return to effort as it is easier to overtake one’s rivals. This leads to higher effort for low and middle ranking

agents. Furthermore, equilibrium utility falls at middle and high ranks and even those with higher endowments can be worse off in the more equal and hence more competitive distribution. However, an increase in the equality of rewards implies there is less difference between a high prize and a low one. This leads to a reduction in incentives and a decrease in equilibrium effort for low and middle ranking competitors, and an increase in their equilibrium utility. Simply put, greater equality of rewards will typically benefit most of society, greater equality of endowments can harm the majority.

In our analysis, we assume formal equality of opportunity. That is, “there is no legal bar to access to education, to all positions and jobs, and that all hiring is meritocratic” (Roemer, 1996, p. 163) and where individuals are rewarded according to their productivity (Lamont, 2003). However, the tournament mechanism of resource allocation allows one to separate the other two forms of inequality – that of endowments and of rewards. Talent could vary widely, but the most talented could receive a monetary reward only slightly greater than the least talented. Alternatively, small differences in talent could lead to big differences in outcomes. This distinction between inequality of endowments and inequality of rewards we believe to be novel.

Why should we assume that success is driven by relative performance? An important reason is empirical. Tournament-like mechanisms are used in practice to determine university admissions, entry into certain professions and promotions and pay within firms. More generally, there is now a significant body of research that suggests that indicators of wellbeing such as job satisfaction (Brown et al., (2004)), health (Marmot et al. (1991), Marmot (2004)) and overall happiness (Easterlin (1974)) are strongly determined by relative position. That is, a highly ranked individual in a poor country can have greater health and happiness than a low ranked individual in a richer country, even though the latter has greater material prosperity. However, this existing literature on relative position and relative concerns has tended to assume that if relative position matters, this implies a distaste for inequality. In particular, Frank (1999, 2000) has argued forcefully that greater inequality exacerbates wasteful social competition. Hopkins and Kornienko (2004) showed that this argument may be problematic in that, in a model with relative concerns, an increase in equality of endowments can make everyone worse off. Here, we provide partial support for Frank’s argument but we emphasize that it is an increase in the inequality of rewards, rather than greater inequality of endowments, that worsens wasteful competition.

We observe that the inequality of rewards has a much better fit with common beliefs about the effects of inequality. An increase in this type of inequality benefits the rich in endowments and hurts the middle-class and poor, who are forced into greater effort. Under some regularity conditions, even stronger welfare effects are possible - namely, that greater inequality of rewards can make all worse off. In contrast, greater inequality of endowments has an opposing effect and can benefit the middle-classes. Of course, this latter finding is in conflict with common intuitions about the role of inequality. What these results suggest in practical terms is that policies to promote equality could have very different effects depending on the form of the intervention. For example, an

attempt to equalize educational achievement, in the sense that it equalizes endowments of those entering the labor market, would have a quite different effect than a policy to equalize incomes, the rewards from labor market activity. The first policy would increase competitive pressure on those in the middle of the range of ability, while the second would reduce it. Thus, once the impact on work effort is included, the first policy surprisingly could have a negative effect on those it was aimed to help, in a sense that it would create pressures on those in the middle to perform more, at the expense of their happiness. In contrast, the second policy would reduce these pressures to perform, increasing wellbeing for those in the middle at the expense of lower performance.

What is crucial in the tournament model we consider is that the shape and the range of the distributions of endowments and rewards themselves determine the marginal return to effort. Consequently, even policy interventions such as lump-sum taxes and transfers can have an impact on incentives if they change either the distributions of endowments or rewards. In fact, there are two distinct effects from any changes in the level of inequality. The first, which we call the direct effect, is simply that under a more equal distribution of endowments or rewards lower ranked individuals will have greater endowments or rewards respectively. However, in either case, there is also the second effect, which we call the incentive or social competitiveness effect. Crucially, the incentive effect of an increase in equality of endowments is positive and opposite of that of an increase in the equality of rewards, which decreases incentives. Note that this incentive effect is created by the competitive externalities present in our tournament model. So, in their absence such as in more conventional neoclassical models, there are only the direct effects so that reward and endowment inequality would appear to have similar results. This may be why the distinction between rewards and endowments has not been made before.

One important assumption of our tournament model is that there is a fixed distribution of indivisible rewards. The justification for this is that in reality there are many desirable things, jobs, places at university, marriage opportunities, that do differ in quality and which are not divisible. A subtle criticism is that even if rewards are indivisible, there might be the possibility of side payments. This possibility is analyzed in a different literature where workers are matched to (indivisible) jobs by an endogenous wage schedule. For example, Costrell and Loury (2004) and Suen (2007) have considered changes in the distribution of ability of workers and in the quality of jobs. There is no incentive effect as there is no choice of effort by workers and all outcomes are Pareto efficient, in distinct contrast to the situation we model. Nonetheless, the shape of the distributions of ability and of jobs affects the distribution of wages. That is, inequality can have a material effect on outcomes even in the presence of side payments. This suggests that what is crucial is simply that rewards in society vary and that some are indivisible and thus positional externalities are present.¹

¹More technically, inequality of endowments and inequality of rewards will have substantial but opposing effects whether matching between competitors and jobs or rewards is done under transferable utility or non-transferable utility.

A further contribution of this paper to the modelling of inequality is its demonstration of the importance of the method of tracking individuals when changes are made in the distribution of endowments. There are two ways of doing this: compare choices and outcomes at a given level of endowment or at a given rank in the distribution of endowments. As Hopkins and Kornienko (2007) point out and as we show here, the two methods apparently give different results: greater equality of endowments leads to higher utility at a given low rank, but lower utility at a given low endowment. But which is the more appropriate comparison? Suppose endowments are intrinsic, for example, talent or ability. Then, for example, an individual with some fixed ability level might observe a change in the distribution of abilities around him as a result of migration. While her own ability is fixed, her rank in the distribution of abilities changes. In contrast, if endowments represent something extrinsic such as wealth, for example, redistribution could change the wealth of many individuals without changing their relative position or rank. Thus, the nature of the initial heterogeneity in endowments suggests whether we should analyze distributional changes from the point of view of a fixed rank or from the point of view of a fixed level of endowment.

The literature on tournaments and contests is extensive. As Konrad (2007) points out in a survey, increased heterogeneity amongst competitors and decreased spread of prizes are both known to reduce equilibrium effort in tournaments.² Our current contribution to this literature is two-fold: the technical contribution here is to consider very general comparative statics for large populations of competitors, while our conceptual contribution is to argue that tournaments help us to understand different forms of social inequality. In work that employs similar techniques, Moldovanu and Sela (2006) consider what would the optimal contest design from the perspective of a contest designer who aimed to maximize either the expected total effort or the expected highest effort from contestants. Hoppe, Moldovanu and Sela (2005) generalize this approach to a two-sided tournament environment similar to the current model. Cole, Mailath and Postlewaite (1992, 1995, 1998) and Fernández and Galí (1999) pioneered the use of rank-order tournament models to study non-market allocation of resources. However, their focus of interest is not inequality but a comparison of the efficiency of different institutions for assigning rewards. While Hopkins and Kornienko (2004, 2007) look at comparative statics arising from changes in the inequality of endowments (with the latter paper contrasting results using indexing by level and by rank) in perfectly discriminating rank-order tournaments, neither paper considers reward inequality.

Finally, we argue that our distinction between endowments and rewards differs from the most common existing concepts of inequality. First, as we argue, inequality of rewards and endowments are logically separate from equality of opportunity. Existing merit-, desert- or effort-based theories of justice assume that those who work more, or have greater merit, should have greater rewards (see Konow (2003) for a survey). However, it seems to have been assumed that, with equality of opportunity, individ-

²Much of this literature concentrates on games in which the mechanism that awards prizes is assumed to be at least partly stochastic. What we model here in a contrast could be called a “perfectly discriminating rank order tournament”.

uals will get their “just” reward. That is, there seems to be little discussion of the fact that the reward schedule could vary even in the presence of equality of opportunity. Second, in the distributive justice literature (see Rawls (1971), Dworkin (1981b), Roemer (1996, 1998) among others) one often encounters the question of equality of “resources” (wealth, but also possibly education or talent). However, these works make no distinction about timing or causation, in the sense that there is no distinction made between what one has initially (endowment) and what one is able to obtain (reward). Third, equality of rewards should not be confused with equality of welfare or equality of outcomes. In this model at least, the welfare of an individual depends jointly on a set of outcomes that includes her endowment, her choice of effort as well as her reward. We discuss the relation of our work to previous literature on inequality in greater detail in Section 7.

2 The Model

In this section, we develop our model, where a large population competes in a tournament-like market to obtain rewards or prizes. We have in mind three prime examples. The first is students competing for places at college. The second is a market for jobs. For example, students in the final year of graduate school seek faculty positions at universities. The third is a marriage market, where singles attempt to attract desirable potential spouses. These three situations are modelled as tournaments by Fernández and Galí (1999), Hopkins (2005) and Cole, Mailath and Postlewaite (1992) respectively. We will use the terminology of “contestants” competing for rewards. Contestants have to make a decision on how to allocate their initial endowment between private consumption and visible performance that acts as a signal of underlying ability. Each contestant is then awarded a reward or prize. These are awarded assortatively with the best performer being awarded the top prize, the median performer the median prize and so on downward with the worst performer receiving the last prize.

We assume a continuum of contestants. They are differentiated in quality with a contestants having differing endowments z with endowments being allocated according to the publicly known distribution $G(z)$ on $[\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$. The level of each contestant’s endowments is her private information. The distribution $G(z)$ is twice differentiable with strictly positive density $g(z)$. A contestant’s level of endowment z has possible specific interpretations such as her wealth or an ability parameter that determines maximum potential performance.³ In particular, contestants must divide their endowments between visible performance x and private consumption or leisure y .

There is also a continuum of prizes or rewards of value s whose publicly known distribution has the twice differentiable distribution function $H(s)$ on $[\underline{s}, \bar{s}]$ and strictly

³For example, suppose all contestants are endowed with the same amount of time that can be used for production or leisure. Then, let z be productivity per hour and a contestant devoting a proportion x/z of time to production will have performance x .

positive density $h(s)$. While the rewards could simply be in cash, this is not necessarily the case. In the context of the academic job market, s could be interpreted as prestige or reputation of a university, in the marriage market, s could be a measure of attractiveness to the opposite sex. After the contestants' choice of performance, rewards will be awarded assortatively, so that the contestant with the highest performance x will gain the prize with highest value \bar{s} . More generally, the rank of the prize awarded will be equal to a contestant's rank in terms of performance.

We have two ideas in mind why rewards might be assigned in such a way. First, such mechanisms are often used in situations such as college admissions to promote a form of equality of opportunity. For example, if z represents ability and x represents academic performance, then the highest rewards go to those of the highest performance which in the equilibrium we consider will be those of highest ability.⁴ Second, the other side of the market could consist of people, potential spouses or employers, rather than inanimate prizes. These potential partners would have to choose between contestants. But it is easy enough to specify suitable preferences for the partners such that the end result in equilibrium would be the same: the best performing contestant obtains the best match.⁵ Here, we assume that such partners are interested in a contestant's performance x mostly in terms of its use as a signal, that is what it reveals about his underlying endowment of ability z .

A contestant's endowment z can be employed in performance x or private consumption $y = z - x$ (that is, the rate of conversion between x and y is normalized to one). The contestants, all have the same utility function

$$U(x, y, s) = U(x, z - x, s). \quad (1)$$

We assume that utility is increasing in all three arguments, performance x , private consumption y and the reward s . That is, there is some private benefit to performance, for example, private satisfaction from studying.⁶ While it is possible to divide one's endowment between x and y , the only way to obtain a reward s is to take part in the tournament.

We assume a series of standard conditions on the utility function that will enable us to derive a monotone equilibrium and clear comparative statics results. (i) U is twice continuously differentiable (smoothness); (ii) $U_x(x, y, s) > 0$, $U_y(x, y, s) > 0$, $U_s(x, y, s) > 0$ (monotonicity); (iii) $U_{xy}(x, y, s) > 0$, $U_{ys}(x, y, s) \geq 0$ and $U_{xs}(x, y, s) \geq 0$ (complementarity); (iv) $U_{ii}(x, y, s) \leq 0$ for $i = x, y, s$ (own concavity); (v) $U_x(x, z - x, s) - U_y(x, z - x, s) = 0$ has a unique solution $x = \gamma(z, s)$ and whenever $x \geq \gamma(z, s)$ it holds that $U_{xs}(x, z - x, s) - U_{ys}(x, z - x, s) \leq 0$. This last condition seems somewhat complicated but it is automatically satisfied if utility is additively or multiplicatively

⁴Fernández and Galí (1999) show that such mechanisms can be more efficient than markets in assigning educational opportunities when capital markets are imperfect.

⁵See Cole, Mailath and Postlewaite (1992, 1998) for explicit consideration of voluntary matching between contestants and potential partners.

⁶Nothing substantial depends on this assumption. All results are qualitatively the same if x has no intrinsic value for contestants.

separable in s . Note also that it implies a competitor would choose a positive performance x even when there are no competitive pressures.

It is natural, perhaps, to think of a competitor's type as her level of endowment. However, given an endowment distribution $G(z)$, an agent with endowment \tilde{z} has rank $\tilde{r} = G(\tilde{z})$ and it is equally valid to think of her type as being \tilde{r} as much as it is \tilde{z} . We have assumed that $G(\cdot)$ is strictly increasing on its support so that there is a one-to-one relation between endowment and rank. There are several advantages of indexing by rank over indexing by endowment level as discussed in detail in Hopkins and Kornienko (2007) and in Section 3 here. Nonetheless, we will use both methods with the analysis with indexing by level of endowment to be found in Section 5. In this section, we will treat each competitor's type as her rank r . Notice that on an agent's endowment can be expressed as a function of his rank or $\tilde{z} = G^{-1}(\tilde{r})$ (i.e. \tilde{z} is at the \tilde{r} -quantile). In particular, let us write $Z(r) = G^{-1}(r)$. Equally, her strategy will be a mapping $x(r) : [0, 1] \rightarrow \mathbf{R}_+$ from rank to performance.

Then, a symmetric equilibrium will be a Nash equilibrium in which all contestants use the same strategy, that is, the same mapping $x(r)$ from rank in endowments to performance. Suppose for the moment that all contestants adopt such a strategy $x(r)$ that, furthermore, is differentiable and strictly increasing (we will go on to show that such an equilibrium exists). Let us aggregate all the performance decisions of the contestants into a distribution summarized by a distribution function $F(x)$. If $x(r)$ is strictly increasing, then there will be no mass points in the distribution of performance, so that $F(x)$ is continuous and strictly increasing. Note that such a strategy is fully separating. One can deduce a contestant's endowments z or his rank in the distribution of endowments r from his choice of performance x .

Prizes will be awarded to contestants in a way that satisfies *competitive equality of opportunity*. Specifically, the way in which rewards are awarded should depend only on an agent's visible performance, x . In contrast, *inequality of opportunity* would be a rule such that the allocation of rewards depends on some further, extraneous factor such as race, age, gender or social status. While such an allocative rule is perhaps more realistic, we leave it aside for the future research as it would imply an additional dimension of inequality among agents, and would call for more advanced methods of analysis.

Given that rewards are indivisible and are ranked from lowest to highest, the obvious way to assign rewards in way that would satisfy competitive equality of opportunity is assortatively: rewards are assigned on the basis of one's rank $F(x)$ in achievement with the highest achievement obtaining the highest reward and so on. This assignment should also be measure-preserving (the equivalent, given a continuum of prizes and contestants, of awarding exactly one prize to each contestant). A possible way to do this is to assign rewards assortatively so that rank in rewards equals rank in endowments, or $H(s) = G(z)$. Note that in a symmetric Nash equilibrium where the strategy $x(r)$ is strictly increasing in an agent's rank, we have that $G(z) = F(x) = r$. That is, an agent's rank r in the distribution of endowments $G(z)$ is equal to her rank in the distribution

of performance. In turn, if rewards are assigned assortatively according to performance so that an agent's rank in the distribution of achievement $F(x)$ is equal to her rank in the distribution of rewards $H(s)$, so that $G(z) = F(x) = H(s) = r$. Then we have an assignment that satisfies equality of opportunity and is measure preserving.⁷

Remark 1 *Competitive equality of opportunity implies that rewards are assigned assortatively based on a competitor's performance x so that the rank of the reward $H(s)$ is equal his/her rank in the distribution of performance $F(x)$. In a fully separating equilibrium, this is equal to his/her rank in endowments so that*

$$G(z) = F(x) = H(s) = r. \quad (2)$$

That implies that, in such an equilibrium, an agent of rank r is allocated a reward $s = H^{-1}(r)$.

Note that this relationship (2) implies that we can define the function

$$S(r) = H^{-1}(r), \quad (3)$$

which gives the equilibrium reward of a contestant of type r , so that $S : [0, 1] \rightarrow [\underline{s}, \bar{s}]$. The marginal increase in reward from an increase in one's rank would be

$$S'(r) = \frac{1}{h(H^{-1}(r))} = \frac{1}{h(S(r))}.$$

This also implies a reduced form utility:

$$U(x, y, s) = U(x(r), Z(r) - x(r), S(r))$$

That is, the tournament with assortative award of prizes implies that each individual's payoffs are increasing in her rank r in the distribution of contestants. It therefore might appear to an outside observer that the individual had some form of social preferences where she cares about her relative position, similar to those analysed by Frank (1985) and Hopkins and Kornienko (2004). As Cole, Mailath and Postlewaite (1992), Postlewaite (1998) point out, this form of tournament therefore gives a strategic basis to such models.

Continuing with the assumption that all agents adopt the same increasing, differentiable strategy $x(r)$, let us see whether any individual agent has an incentive to deviate. Suppose that instead of following the strategy followed by the others, an agent with rank r chooses $x_i = x(\tilde{r})$, that is, she chooses performance as though she had rank \tilde{r} . Note that her utility would be equal to

$$U = U(x(\tilde{r}), Z(r) - x(\tilde{r}), S(\tilde{r})).$$

⁷Notice that our formulation of equality of opportunity is consistent with Roemer (1996, 1998), as here all individuals of the same type - defined by the individual's endowment - submit the same effort (so that the distribution of effort within their own type is degenerate) and get the same reward.

We differentiate this with respect to \tilde{r} . Then, given that in a symmetric equilibrium, the agent uses the equilibrium strategy and so $\tilde{r} = r$, this gives the first order condition,

$$x'(r) (U_x(x, Z(r) - x, S(r)) - U_y(x, Z(r) - x, S(r))) + U_s(x, Z(r) - x, S(r))S'(r) = 0. \quad (4)$$

This first order condition balances disutility from increasing effort x against the implied marginal benefit in terms of an increased reward from doing so. It defines a differential equation,

$$x'(r) = \frac{U_s(x, Z(r) - x, S(r))}{U_y(x, Z(r) - x, S(r)) - U_x(x, Z(r) - x, S(r))} S'(r) = \phi(x, Z(r), S(r))S'(r). \quad (5)$$

An important point to recognize is that this differential equation and the equilibrium strategy, which is its solution, both depend on the distribution of endowments through $Z(r) = G^{-1}(r)$ and the distribution of rewards through $S(r) = H^{-1}(r)$.

Our next step is to specify what Frank (1985) and Hopkins and Kornienko (2004) call the “cooperative choice”, which is the optimal consumption choice $(x_c(r), y_c(r))$ when an individual does not or cannot affect her reward. Specifically, assume that an agent of rank r is simply assigned a reward $S(r)$ rather than having to compete for it. Her optimal choice in these circumstances must satisfy the standard marginal condition

$$U_x(x, Z(r) - x, S(r)) - U_y(x, Z(r) - x, S(r)) = 0. \quad (6)$$

By assumption (v) above, there is a solution $x_c(r) = \gamma(Z(r), S(r))$ to this maximization problem. The cooperative strategy also enables us to fix the appropriate boundary condition for the differential equation (5). Thus, the initial condition, or the choice of the individual with the lowest rank zero is,

$$x(0) = x_c(0). \quad (7)$$

We can now show the following existence result. It shows that there is only one fully separating equilibrium. Specifically, if all contestants adopt a strategy $x(r)$ that is the solution to the above differential equation (5) with boundary condition (7) and rewards are awarded assortatively according to the rule (2), then no contestant has an incentive to deviate. Further, as this solution $x(r)$ is necessarily strictly increasing, it is fully separating with contestants with high endowments producing a high level of performance. Thus, an authority organizing the tournament to promote equality of opportunity would be rational to give high rewards to high performers as high performance signifies high ability. Or, in the matching story, potential partners should prefer to match with high performers. Note, however, this will typically not be the only equilibrium. As is common in signalling models, other equilibria such as pooling equilibria will exist. In this paper, we concentrate on the separating equilibrium as this seems the most natural for the settings we consider.

Proposition 1 *The differential equation (5) with boundary conditions (7) has a unique solution which is the only symmetric separating equilibrium of the tournament. Equilibrium performance $x(r)$ is greater than cooperative amount, that is $x(r) > x_c(r)$ on $(0, 1]$.*

This implies that the cooperative outcome $x_c(r)$ Pareto dominates the equilibrium performance $x(r)$ from the point of view of the contestants. As is common in competitive situations, the contestants could make themselves all better off by agreeing to work less. How much more will depend on the exact form of the equilibrium strategy $x(r)$ which in turn depends on the distribution of endowments and the distribution of rewards. We will go on to look at how equilibrium choices and welfare change in response to changes in these distributions.

Note that this welfare result holds even though contestants derive utility from their own performance, that is, it is not a pure signal. However, if other parties, for example, partners or employers, also benefit from the competitors' efforts, overall welfare judgements are potentially more complicated. Hopkins (2005) looks at this issue and finds that in the presence of incomplete information, the level of performance can be excessive even considering the welfare of employers. However, it is clearly true that if their performance is sufficiently valuable to society, then the equilibrium performance level could be excessively low relative to the social optimum even if it is too high from the perspective of the competitors. Another possibility is that, like in Cole, Mailath and Postlewaite's (1992) original tournament model, the beneficiaries are the next generation. In this case, social competition leads to a growth rate that is higher than the present generation would choose (see also Corneo and Jeanne (1997)).

3 Two Effects of Changes in Equality

Since, to our knowledge, the effects of the distributions of endowments and rewards in rank-based tournaments have been little explored, in this section we introduce the intuition behind our analysis of how changes in either the distribution of endowments or in the distribution of rewards affect individuals. We make the point that in both cases a change influences individual welfare through two channels, a direct effect and an incentive effect. It is the second effect which is unique to our tournament model in that here, in contrast to standard models, changes in the endowment or rewards of others will change the incentives of individuals to engage in effort. But even the direct effect is not straightforward as whether it is positive or negative can depend on the way it is viewed, whether from a position of a constant endowment or from a constant rank. These differing effects we now try to make clear in a simple way before moving to formal results in the next section.

Consider first a situation where individuals differ in their natural endowments, such as talent, ability, physical attractiveness, and so on. Then, while the distribution of

endowments may change, through immigration for example, the endowment of an individual will stay the same. However, if the distribution does change, then typically the rank of such an individual will change even if her endowment does not. In such case, it makes sense to fix an individual by the *level* of her endowment z , and consider what happens as her *rank* r changes in response to changes in the distribution.

Consider instead a situation where individual endowments are in terms of income, wealth, capital goods, and so on. In this case, an individual's endowment is not intrinsic and could be changed. For example, a redistributive tax policy could change the endowments of most (if not all) individuals. In such situations, it is common among policy analysts to analyze policy consequences at different ranks - for example, for the median individual or for lower and upper quartiles. Effectively in such analysis, one fixes an individual by her *rank* in the distribution of endowments r , but allows for her *endowment* z to change as the distribution of endowments varies.⁸

In other words, different forms of endowments call for different methods of analysis - one that holds constant an individual's endowment z , and another that fixes an individual's rank r . Notice that given our assumption of strictly increasing and continuous distribution of endowments $G(z)$, one's endowment and one's rank have a one-to-one correspondence. The two methods, however, differ in their analytical properties. We find the rank method to be easier. It allows less restrictive comparative statics analysis as one does not have to maintain the same support of the distribution of either rewards or endowments. However, it requires the use of stochastic orders which work in terms of rank rather than in terms of levels but which are not well known - namely the dispersion order and the star order (both of these orders are presented in Appendix A). In contrast, the endowment method is more difficult analytically, but it requires better known stochastic orders - such as refinements of standard first- and second-order stochastic dominance.

The distinction among rank-indexing and level-indexing is very important for the understanding of the effects of changes in inequality. Not only do the two indexing methods require different comparative statics methods, they also differ in how change in inequality is channelled into individual choices and well-being, as we will now see.⁹

In what follows we assume that there is a change in either the distributions of endowments or in the distributions of rewards, but not both. That is, we do not change both distributions at once. We label the initial distribution a for *ex ante* and the changed distribution p for *ex post*. We will consider two regimes. In regime G , we assume that the societies have identical distributions of rewards $H_a = H_p = H$ but differ in the distribution of endowments $G_a \neq G_p$. In regime H , we assume that the

⁸When redistributive policies are rank-preserving (such as a policy combining a lump-sum transfer and a proportional tax), analysis at a fixed rank is equivalent doing analysis for a given individual before and after the change.

⁹The same issues arise in assignment models. For example, Costrell and Loury (2004) use what we call rank indexing and Suen (2007) uses level indexing for comparative statics and obtain apparently different results.

societies have identical distributions of endowments, that is $G_a = G_p = G$, but differ in the distributions of rewards, i.e. $H_a \neq H_p$.

We go on to show how, given equality of opportunity, changes in the inequality of endowments and rewards affect different individuals. We distinguish between two different consequences of changes in the level of inequality, which we call the direct effect and the incentive effect.

3.1 The Direct Effect

The *direct effect* simply arises because changes in the social or economic environment of an individual have direct consequences on that individual's choices and well-being - as they will change her endowment z , or her rank r , and/or her reward s - these direct consequences will vary with the indexing method.

Notice first that different endowment distributions imply that almost all individuals with fixed rank r have different endowments in the two societies, i.e. $Z_a = G_a^{-1}(r) \neq G_p^{-1}(r) = Z_p$ - even though their equilibrium reward $S = H^{-1}(r)$ does not change (see Figure 1). In contrast, almost all individuals with fixed endowment z have different ranks in the two societies, i.e. $r_a = G_a(z) \neq G_p(z) = r_p$, and thus different equilibrium rewards $S_a = H^{-1}(G_a(z)) \neq H^{-1}(G_p(z)) = S_p$ (see Figure 2).

An easy way to understand the differences between the two perspectives is to compare Figures 1 and 2, which show similar changes in the distribution of endowments. In both cases, the ex post distribution G_p is more equal than the original distribution G_a . As illustrated in Figure 2, for a fixed level of endowments z_1 , in the more equal distribution of endowments a low ranked agent will have a lower reward. That is, the direct effect of redistribution is *negative* for low-ranked agents under indexing by endowment levels. However, in Figure 1, it is shown that for a fixed rank r_1 a low ranked agent will have the same reward but a higher level of endowments in a more equal distribution of rewards, the direct effect of redistribution for the low ranked is *positive*. Comparisons at a fixed level of endowment or at a fixed rank give a very different view of the same phenomenon.

In contrast, when we change the distribution of rewards, the direct effect does not depend on whether we index by rank or by level. The effect of redistribution of rewards will be positive for the low ranked. For example, see Figure 3 where now the ex post distribution of rewards H_p is more equal than the ex ante distribution $H_a(s)$. We have $S_a = H_a^{-1}(r_1) = H_a^{-1}(G(z_1)) < H_p^{-1}(r_1) = H_p^{-1}(G(z_1)) = S_p$. One can also see that it will be negative for the high-ranked.

Remark 2 *The direct effect of greater equality can be summarized as follows.*

(i) *Consider first rank-indexing. Suppose endowments become more equal, then, in equilibrium, low (high) ranking agents have higher (lower) endowments. Suppose,*

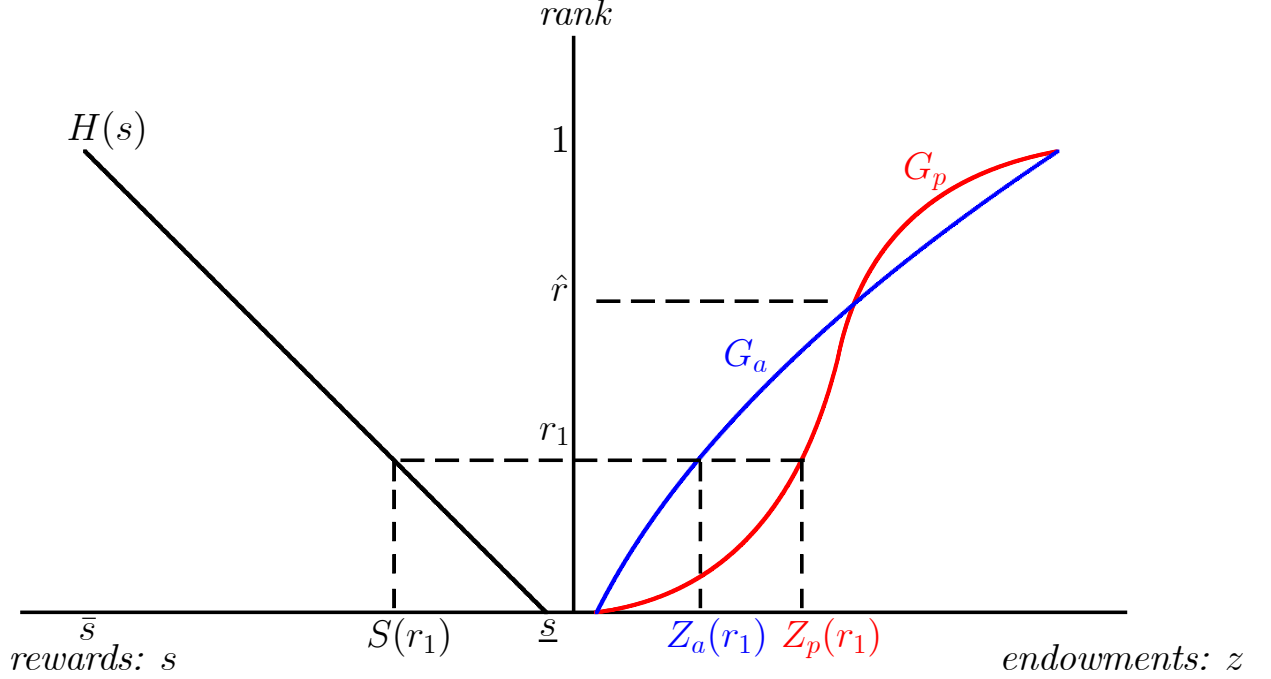


Figure 1: The direct effect in Regime G - under rank-indexing: a contestant with low rank r_1 has a higher endowment $Z_p(r_1)$ under the more equal distribution of endowments G_p than the endowment $Z_a(r_1)$ under the less equal distribution of endowments G_a , and in both cases has a reward $S(r_1)$.

instead, rewards become more equal, then, in equilibrium, low (high) ranking agents also have higher (lower) rewards.

(ii) Consider now level-indexing. Suppose endowments become more equal, then, in equilibrium, low (high) ranking agents have lower (higher) endowments. Suppose, instead, rewards become more equal, then, in equilibrium, low (high) ranking agents, in contrast, have higher (lower) rewards.

Note that the direct effect is what one would obtain under classical assumptions. To see this, suppose rewards were assigned non-competitively by a social planner according to one's rank in the endowment distribution, i.e. $H(s) = G(z)$. Then choices would be made non-competitively, corresponding to the "cooperative" choices as set out in Section 2. The direct effect of changes in distributions on individual choices and welfare will thus be a consequence of a direct response to changes in agent's endowment z or in her reward s . Furthermore, if we use the rank indexing, the direct effect of greater equality of rewards is qualitatively similar to greater equality of endowments. Very simply, it benefits the low ranked at the cost of the high ranked which is not surprising. However, this makes the point that under classical assumptions (that is, without positional or competitive externalities in choices), equality of rewards and endowments are indistinguishable, which may explain why this distinction has not been made before. Though, note that even here, the direct effects are not the same under level indexing.

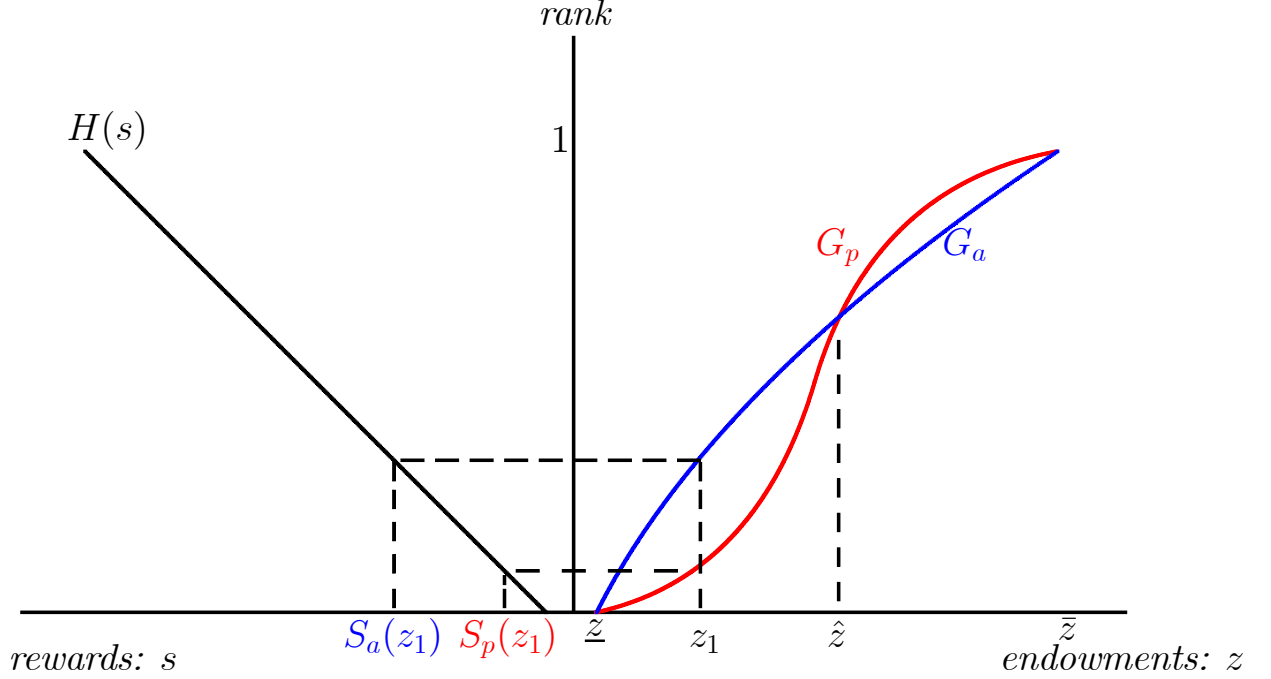


Figure 2: The direct effect in Regime G - under level-indexing: a contestant with fixed low endowment z_1 has a reward $S_p(z_1)$ under the more equal distribution of endowments G_p that is worse than the reward $S_a(z_1)$ under the less equal distribution of endowments G_a .

3.2 The Incentive Effect

Now let us turn to the *incentive* (or marginal, positional, strategic, or social competitiveness) effect of changes in inequality. Importantly, the effect of less dispersion in rewards and endowments have an opposite incentive effect regardless of the indexing method used. The incentive effect is the result of agents' strategic interactions. As was shown in Hopkins and Kornienko (2004, 2007), in the non-cooperative game where agent's rank matters for her welfare, the "social density", or "social competitiveness", is important as it changes incentives. The incentive effect of changes in distributions on individual choices and welfare will depends largely on the densities (or marginal frequencies) of endowments and rewards, $g(z)$ and $h(s)$. This incentive effect can be modelled using the dispersion order (presented in Appendix A) which is a stochastic order used to compare distributions in terms of their densities.

Remark 3 *The incentive effect of greater equality can be summarized as follows.*

(i) *Suppose endowments become less dispersed, then there is an increase in the marginal return to effort, as it is now easier to surpass neighbors, so that agents tend to increase their effort.*

(ii) *Suppose rewards become less dispersed, then there is a decrease in the marginal*

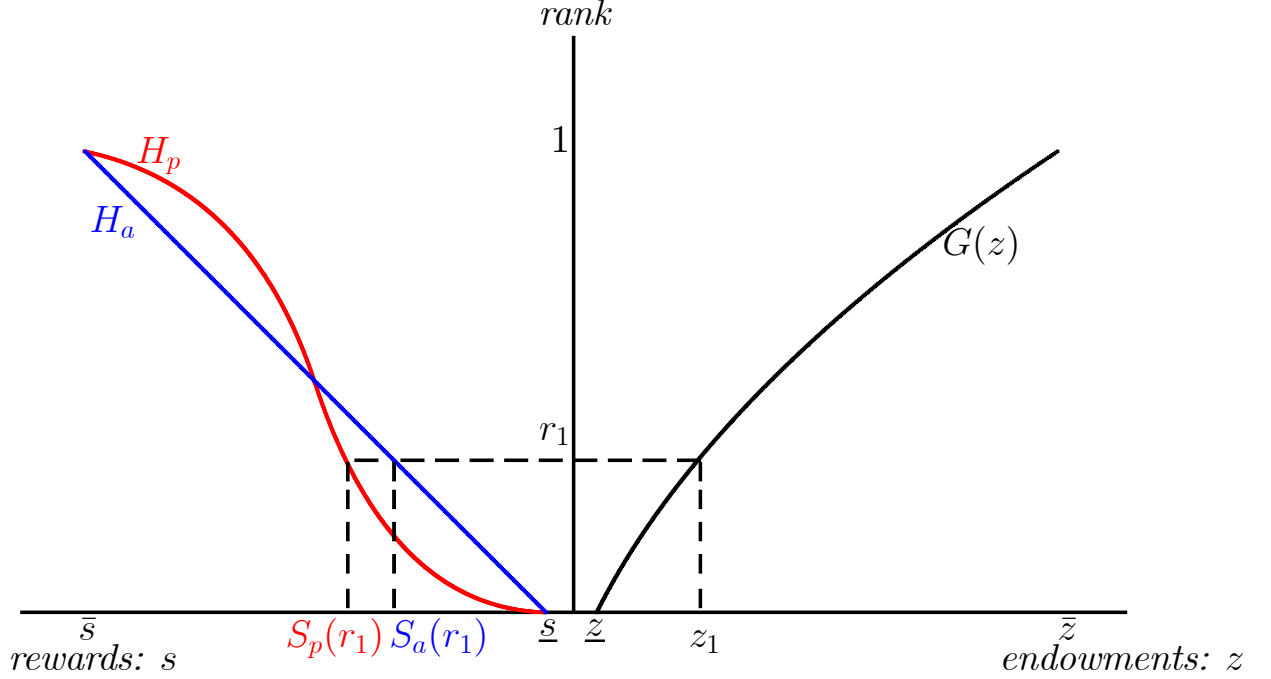


Figure 3: The direct effect in Regime H - under rank- and level-indexing: a contestant with low rank r_1 has higher reward $S_p(r_1)$ under the more equal distribution of rewards H_p than reward $S_a(r_1)$ under the less equal distribution of rewards S_a .

return to effort as rewards are now more similar, so that agents tend to decrease their effort.

To find the *total effect*, which includes both direct and incentive effects, one needs to analyze how changes in inequality affects behavior, which we turn to now.

4 Effects of Changing Inequality Under Indexing By Rank

We will now consider the effect on equilibrium utility and strategies of changes in the distribution of endowments $G(z)$ and changes in the distribution of rewards $H(s)$. In this section, we do this by comparing behavior before and after the change at each rank in the distribution of endowments, using the rank indexing methodology as discussed in Section 3. We saw in Section 2 that equilibrium behavior depends on the reward function S which is jointly determined by G and H . Thus, as the distribution of endowments G or the distribution of rewards H (or both) change, so does the reward function S . Thus, a change in either distribution of endowments or rewards (or both) translates into a change in equilibrium choice of performance $x(r)$ and, thus, into a change in individual welfare. We have just seen in the previous section in intuition as

to why changes in the level of equality of endowments and rewards can have different effects.

Equilibrium utility in terms of rank will be $U(r) = U(x(r), Z(r) - x(r), S(r))$. By the envelope theorem we have

$$U'(r) = \frac{U_y(x(r), Z(r) - x(r), S(r))}{g(Z(r))} \quad (8)$$

We are interested in what effect a change in the level of inequality would have on equilibrium utility and hence individual welfare. Note though that as average utility is $\int U(r)dr$, if individual welfare $U(r)$ rises at every rank then social welfare will surely rise.

In what follows we assume that there is a change in either the distributions of endowments or in the distributions of rewards, but not both. That is, we do not change both distributions at once. We label the initial distribution a for *ex ante* and the changed distribution p for *ex post*. We will consider two regimes - one focusing on the effects of changes in distribution of endowments (Regime G) and the one focusing on the effects of changes in distribution of rewards (Regime H). In doing this, we make use of the dispersion order, which as the name suggests, is a way of ordering distributions in terms of their dispersion. Please see Appendix A for details. Our results with respect to inequality of endowments are a generalization of those in Hopkins and Kornienko (2007), who examine a similar problem with a less general specification of utility but who do not consider reward inequality.

4.1 Change in Endowments (Regime G)

We investigate in this section the effects of changes in the distribution of endowments on equilibrium performance decisions and equilibrium utility. In particular, we find that an increase in the equality of endowments can have adverse effects. This is because as peoples' endowments become closer together, it is easier to overtake one's neighbors. This leads to a general increase in social competition. While redistribution can benefit those who receive higher endowments, even some of these will be worse off as a consequence of greater competition.

In regime G , we assume that the societies have identical distributions of rewards, i.e. $H_a = H_p = H$, but differ in the distributions of endowments, i.e. $G_a \neq G_p$ and in fact are distinct, that is, equal at only a finite number of points. Different endowments imply that the two societies have different endowment functions, i.e. $Z_a = G_a^{-1}(r)$ and $Z_p = G_p^{-1}(r)$.

Our first result is to show that if a range of contestants receive an increase in endowments, they will respond with higher performance.

Proposition 2 *Suppose that endowments are higher ex post so that $Z_p(r) \geq Z_a(r)$ on an interval $[0, \hat{r}]$ where \hat{r} is the point of first crossing of $Z_p(r)$ and $Z_a(r)$. Then $x_p(0) \geq x_a(0)$ and ex post performance is higher on that interval: $x_p(r) > x_a(r)$ on $(0, \hat{r}]$.*

A consequence of this is that if the new distribution of endowments G_p stochastically dominates the old, then performance will be higher for all agents. Note that if G_p stochastically dominates G_a then by definition $G_p(z) \leq G_a(z)$ for all z , which in turn implies that $Z_p(r) \geq Z_a(r)$ for all $r \in [0, 1]$. That is, in a richer society where endowments are higher for every agent, performance is higher for all.

Corollary 1 *Suppose that endowments are stochastically higher ex post so that $Z_p(r) \geq Z_a(r)$ for all $r \in [0, 1]$, then performance rises almost everywhere: $x_p(r) > x_a(r)$ on $(0, 1]$.*

We can now give a sufficient condition for equilibrium utility to rise for all agents and hence for an increase in social welfare. The condition has two parts. First, endowments must be more dispersed in the sense of the dispersion order or $G_p \geq_d G_a$ (see Appendix A for the definition and properties of this and subsequently used stochastic orders). Second, the lowest ranked agent must be no worse off or $Z_p(0) \geq Z_a(0)$. The point is that, as utility both depends on endowments and the degree of social competition, one can guarantee an increase in endowments will lead to an increase in utility if at the same time the social density does not rise.

Proposition 3 *Suppose endowments are more dispersed ex post $G_p \geq_d G_a$ and minimum endowments no lower $Z_p(0) \geq Z_a(0)$, then utility is higher ex post almost everywhere: $U_p(r) > U_a(r)$ on $(0, 1]$.*

Our final result in this subsection concerns an increase in equality. As remarked, there are two resulting effects. Figure 1 illustrates the direct effect: with a more equal distribution of endowments, the low ranked have higher endowments ex post. However, as we have argued, the marginal effect works toward greater competition. As people are closer together, it is easier to overtake one's neighbors. For the low ranked, the direct effect dominates. For the middle class, the marginal effect is more important, whereas for the upper classes, they lose both from redistribution and from greater competition. We thus find that the middle and upper classes are worse off. This is illustrated in Figure 4.

Specifically, we suppose the distribution of endowments becomes less dispersed in terms of the dispersion order. Furthermore, the lowest ranked agent has more endowments, or $Z_p(0) > Z_a(0)$, and the highest ranked has less $Z_p(1) < Z_a(1)$. Thus, in a clear sense the distribution G_p of endowments is more equal than distribution G_a .

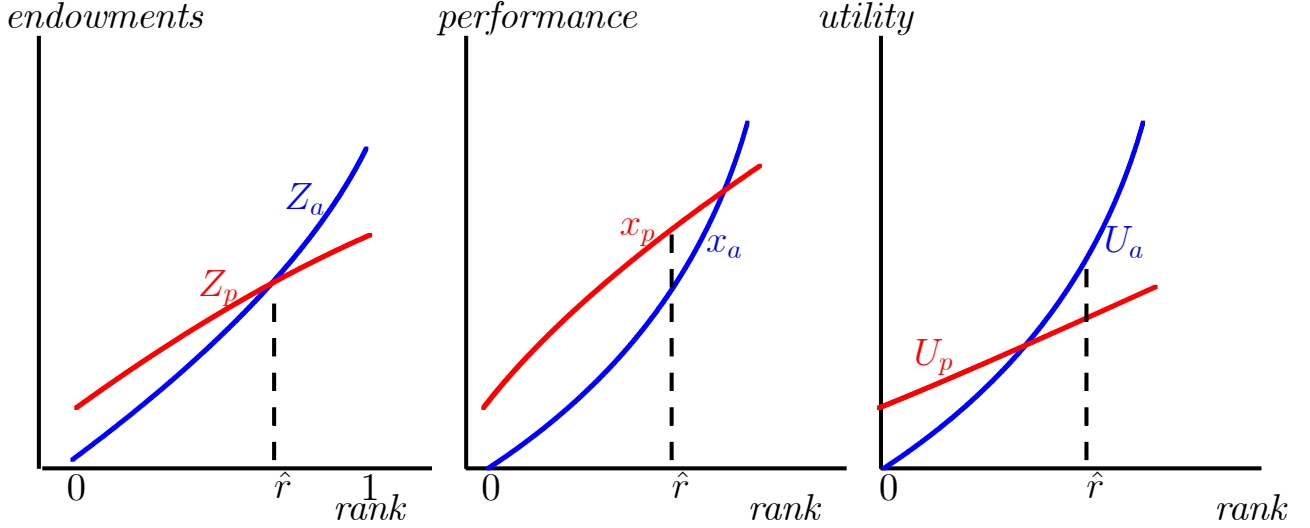


Figure 4: Greater equality of endowments: typical comparative statics when ex post endowments Z_p are more equal than ex ante Z_a (indexing by rank). Performance rises at lower and middle ranks; but utility falls at middle and upper ranks.

Proposition 4 *Suppose that the minimum level of endowments is higher ex post*

$$Z_p(0) > Z_a(0) \quad (9)$$

and endowments are less dispersed ex post

$$g_p(Z_p(r)) \geq g_a(Z_a(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_a \geq_d G_p \quad (10)$$

and also suppose that the maximum level of endowments is lower ex post

$$Z_p(1) < Z_a(1) \quad (11)$$

Then, performance is higher ex post for the bottom and middle: $x_p(r) > x_a(r)$ on $[0, \hat{r}]$ where \hat{r} is the only point of crossing of $Z_a(r)$ and $Z_p(r)$. Second, utility rises at the bottom, $U_p(0) > U_a(0)$, but utility is lower ex post for the middle and top, $U_p(r) < U_a(r)$ for all $r \in [\hat{r}, 1]$.

Note that this result implies that there are middle ranking agents who are worse off even though they have higher endowments ex post (again see Figure 4 for the outcomes for individuals just to the left of \hat{r}). However, the effect at the relatively low ranked individuals, i.e. those with $r \in (0, \hat{r})$ is, in general, ambiguous.

4.2 Changes in Rewards (Regime H)

In this subsection, we find that the effects of changes in rewards are quite different from those arising from changes in endowments. The first point is that the effect of greater

equality in rewards has the opposite marginal effect to greater equality in endowments. Greater equality of rewards implies that the marginal return to greater effort is relatively low, and will tend to reduce competition. This will tend to make competitors better off. However, for high ranking competitors who expect high rewards, the effect is ambiguous. In a more equal society they work less hard but obtain lower rewards.

In regime H , we assume that the societies have identical distributions of endowments, i.e. $G_a = G_p = G$, but differ in the distributions of rewards, i.e. $H_a \neq H_p$ and in fact are distinct, that is, equal at only a finite number of points.. Again, different rewards imply that the two societies have also different reward functions, i.e. $S_a(r) = H_a^{-1}(r)$ and $S_p(r) = H_p^{-1}(r)$.

Our first result concerns sufficient conditions for greater effort by all competitors. We find that if rewards are lower at every rank and the rewards are more dispersed, then the environment is definitely more competitive and effort rises at every rank.

Proposition 5 *Suppose that the rewards are more dispersed ex post*

$$S'_p(r) \geq S'_a(r) \text{ on } (0, 1) \Leftrightarrow h_p(S_p(r)) \leq h_a(S_a(r)) \text{ on } (0, 1) \Leftrightarrow H_p \geq_d H_a \quad (12)$$

and that the minimum reward is lower ex post

$$S_p(0) < S_a(0) \quad (13)$$

and then performance is higher ex post so that $x_p(r) > x_a(r)$ on $(0, \hat{r}]$ where \hat{r} is the first crossing point of $S_p(r)$ and $S_a(r)$.

This leads to the following corollary. If rewards are more unequal and lower at every rank, then performance increases for every agent.

Corollary 2 *Suppose that the ex-post rewards are more dispersed and also are stochastically lower, i.e. $H_p \geq_d H_a$ and $S_p(r) \leq S_a(r)$ for all $r \in [0, 1]$, then performance rises almost everywhere: $x_p(r) > x_a(r)$ on $(0, 1]$.*

Note that if one makes stronger assumptions on the utility function, one can still obtain an increase in performance at all ranks without the stochastic dominance assumption of Corollary 2. First we look if utility is additively separable in rewards.

Proposition 6 *Assume utility is additively separable in rewards, that is $U = V(x, y) + s$ for some function V such that conditions (i) to (v) on U are still satisfied, then if $H_p \geq_d H_a$, it follows that $x_p(r) > x_a(r)$ almost everywhere on $[0, 1]$.*

We can obtain a similar result if utility is multiplicatively separable in rewards. We use the star order that is defined and discussed in detail in Appendix A. But, more informally, the star order implies that H_p is more dispersed or stochastically lower than H_a but not necessarily both as we assume in Corollary 2.

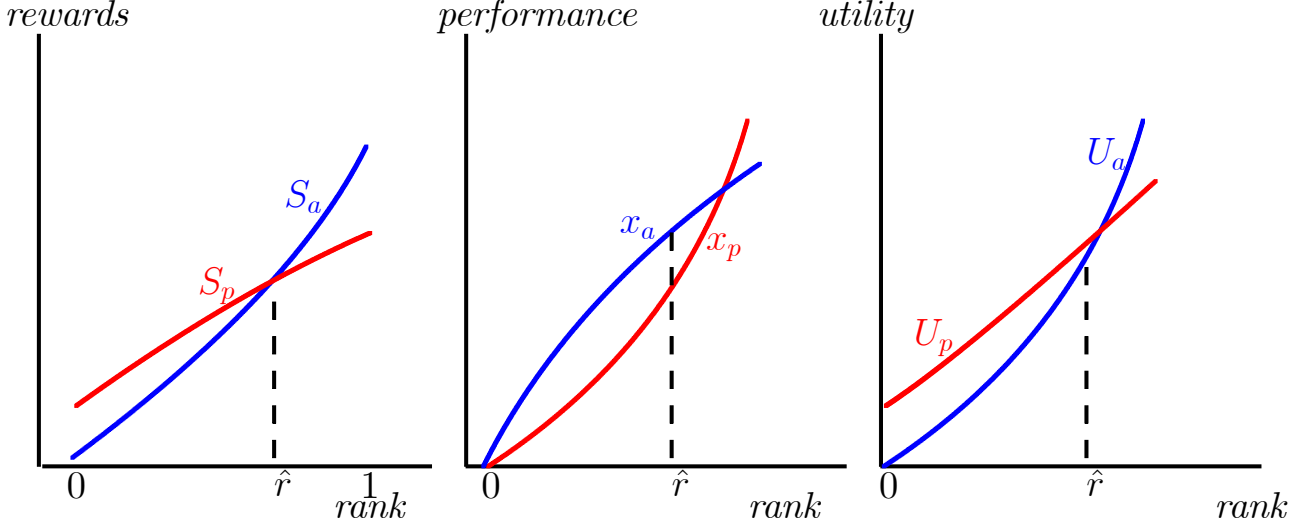


Figure 5: Greater equality of rewards: typical comparative statics when ex post rewards S_p are more equal than ex ante S_a (indexing by rank). Performance falls and utility rises at low and middle ranks.

Proposition 7 *If rewards are multiplicatively separable or $U = V(x, y)$ s for some function V such that conditions (i) to (v) on U are still satisfied, then if $H_p \geq_* H_a$, H_p is more dispersed in the star order, it follows that $x_p(r) > x_a(r)$ almost everywhere on $[0, 1]$.*

We next identify a sufficient condition for an increase in equilibrium utility at every rank. This is much simpler than when considering changes in the distribution of endowments. Here, we simply require that the new distribution H_p stochastically dominates the old H_a and that the lowest reward $S_p(0)$ is strictly higher. This implies that $S_p(r) \geq S_a(r)$ for all r , or rewards are higher at every rank. As this will also decrease the incentives to compete, it is not surprising that equilibrium utility rises.

Proposition 8 *If the minimum reward is higher ex post $S_p(0) > S_a(0)$ and rewards are everywhere else no lower, $S_p(r) \geq S_a(r)$ for all $r \in (0, 1]$, then utility is everywhere higher ex post: $U_p(r) > U_a(r)$ on $[0, 1]$.*

We turn to inequality. As illustrated in Figure 3, the direct effect of greater equality in rewards benefits the low-ranked simply because their rewards will typically be higher. However, we can identify another effect. The compression of rewards will decrease the marginal incentive to compete and performance will fall. This will further benefit competitors. Thus, as we see in Figure 5, utility will rise even for agent with rank \hat{r} whose reward is unchanged.

Proposition 9 *Suppose that the lowest reward is higher ex post*

$$S_p(0) > S_a(0) \tag{14}$$

and also rewards are less dispersed ex post

$$S'_p(r) \leq S'_a(r) \text{ for all } r \in (0, 1) \Leftrightarrow H_a \geq_d H_p \quad (15)$$

and also suppose that the highest reward is lower ex post

$$S_p(1) < S_a(1). \quad (16)$$

Then performance is lower ex post $x_p(r) < x_a(r)$ on $(0, \hat{r}]$ where \hat{r} is the only point of crossing of $S_a(r)$ and $S_p(r)$. Second, utility is higher on that interval: $U_p(r) > U_a(r)$ for all $r \in [0, \hat{r}]$.

We have already seen, Propositions 6 and 7, that in some special cases, greater dispersion of rewards is sufficient to make performance rise for all competitors. We give an example of this, which has another interesting property.

Example 1 Suppose $U(x, y, s) = x^\alpha y s$ for some $\alpha < 1$, so rewards are multiplicatively separable. Assume that endowments are uniform on $[1, 2]$. Then if, for example, rewards go from being uniform on $[0.5, 2.5]$ ($H_a = 0.5s - 0.25$ or $S_a = 2r + 0.5$) to being uniform on $[1, 2]$ ($H_p = s - 1$ or $S_p = r + 1$) then by Proposition 7, performance must fall almost everywhere as these two distributions satisfy $H_p \leq_* H_a$, the ex post distribution is less dispersed in terms of the star order (and, also, the dispersion order). Note that the lowest competitor would have a higher utility under the ex post distribution, i.e. $U_p(0) > U_a(0)$, as she has a higher reward (but the same endowment). Indeed, everyone with rank up to 0.5 must be better off by Proposition 9 as here the crossing point of S_a and S_p is 0.5. But, further, here $U'(r) = x^\alpha(r) Z'(r) S(r)$. If α is reasonably low so that the influence of the lower performance ex post is not large, the slope of utility in rank will not be very different ex post. Thus, for example, for $\alpha < 0.35$, everyone will be better off under the less dispersed distribution H_p .

That is, it is possible by making rewards less dispersed to reduce total performance but make a Pareto improvement. Everyone will be happier because everyone works less. This raises the question as to whether it would be possible to make everyone better off by altering the level of inequality of endowments. However, while a greater dispersion of endowments by Proposition 4 reduces performance for most (and possibly all) competitors, it cannot make all better off for a fixed mean endowment. This is because the greater dispersion would lower the utility of a low ranked competitors, as they would have a lower endowment in the more dispersed distribution.

5 Results under Indexing by Level of Endowment

We now consider a situation where the endowment is intrinsic to the agent, for example, talent. We, therefore, use the level-indexing method and compare an agent's utility

before and after changes in the level of inequality given this fixed level of endowment. As this method has been used before, for example by Hopkins and Kornienko (2004) and Hopkins (2005)), it thus requires less extensive coverage. We find an apparently different outcome from that under rank indexing as those with low endowments are now worse off under greater equality of endowments. The reason for this is that, as discussed in Section 3, the direct effect of greater equality on an individual on a fixed low level of endowments is negative, as opposed to positive under rank indexing.

We now look at the tournament from the perspective of indexing by levels of endowments. That is, we consider the model introduced in Section 2 in terms of endowments z not rank r . As before a continuum of contestants choose x to maximize utility (1). Given the assortative assignment of rewards (2), we can now write the equilibrium reward as a function of endowment as $S(z) = H^{-1}(G(z))$. We look for a strictly increasing symmetric equilibrium strategy as a function of endowments. The equilibrium strategy $x(z)$ will be a solution to the following differential equation, compare equation (5),

$$\frac{dx(z)}{dz} = \frac{U_s(x, z - x, S(z))g(z)}{U_y(x, z - x, S(z)) - U_x(x, z - x, S(z))h(S(z))} = \frac{dx(r)}{dr} \frac{dr}{dz} = \frac{dx(r)}{dr} g(z). \quad (17)$$

The boundary condition will be $x(\underline{z}) = x_c(G(\underline{z}))$, that is the same as in rank terms (7). The only separating equilibrium in terms of endowments $x(z)$ will be a solution to the above equation. This is a direct consequence of Proposition 1. Working in terms of endowments or ranks does not change the underlying game or its equilibria. We emphasize that they are just different ways of looking at the same behavior.

We will also look at individual welfare in terms of endowments. Define $U(z) = U(x(z), z - x(z), S(z))$, that is $U(z)$ is equilibrium utility in terms of endowments z . We show that an increase in equality of endowments amongst competitors reduces the utility of the weakest competitors. In contrast, a similar decrease in the dispersion of the rewards has an opposite effect. In contrast to our work using rank-indexing, we assume here that G_a and G_p have the same support $[\underline{z}, \bar{z}]$ and that similarly there is a common support $[\underline{s}, \bar{s}]$ for the distributions of rewards H_a and H_p . Here we use second order stochastic dominance to order distributions in terms of dispersion (see Appendix A for the relationship among different stochastic orders).

Proposition 10 *Let $U_a(z)$ and $U_p(z)$ be the equilibrium utilities in terms of endowments ex ante and ex post respectively.*

(i) *Suppose that G_p second order stochastically dominates G_a . Denote the first crossing of $G_a(z)$ and $G_p(z)$ as \hat{z} . Then, utility falls for the bottom and middle $U_p(z) \leq U_a(z)$ for all $z \in [\underline{z}, \hat{z}]$.*

(ii) *Suppose that H_p second order stochastically dominates H_a . Denote the first crossing of $H_a(s)$ and $H_p(s)$ as \hat{s} , and denote $\hat{z} = S^{-1}(\hat{s}) = G^{-1}(H_p(\hat{s})) = G^{-1}(H_a(\hat{s}))$. Then, utility rises for the bottom and middle $U_p(z) \geq U_a(z)$ for all $z \in [\underline{z}, \hat{z}]$.*

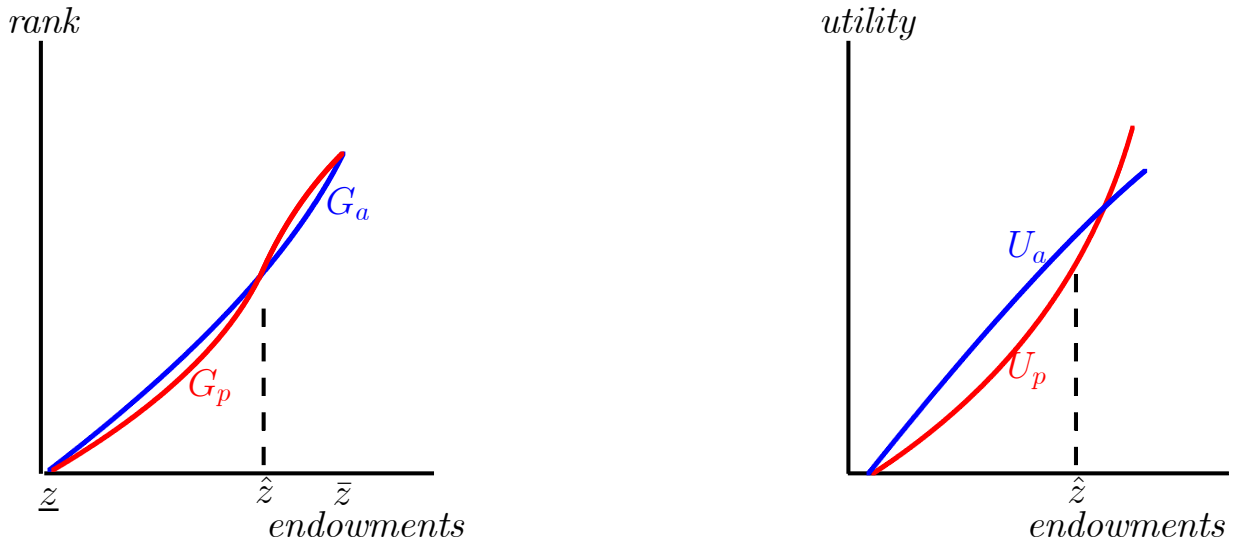


Figure 6: Greater equality of endowments: typical comparative statics when the ex post distribution of endowments G_p is more equal than ex ante G_a (indexing by levels). Utility falls at low and middle levels of endowments.

That is, for those with relatively low endowments, that is, for those whose endowment is less than \hat{z} (see Figure 6), a more equal distribution of endowments leads to lower individual welfare, while, conversely, a similar decrease in inequality of rewards results in an increase in individual welfare. This is because, as discussed in Section 3, the direct effect of greater equality on an individual on a fixed low level of endowments is negative, in that she will now have a lower reward (again see Figure 2). This is because with the reduction in inequality there are more contestants with middling endowments who will now take the middling rewards. Contestants with a fixed low endowment will now receive a lower reward. The incentive to compete is also increased by the greater social density and so even those in the middle will be worse off as they compete harder. Conversely, the direct effect of more equal rewards is positive and incentives to compete are reduced.

6 Summary of the Results

The main question of our paper is as follows. Suppose individuals differ only in their starting endowments (ability, wealth). Suppose individuals compete with others for rewards (college places, jobs, marriage partners, political rents) by taking a visible action (make an effort, produce output). Suppose rewards to success are allocated via a tournament. Suppose competitive equality of opportunity holds - so that rewards are assigned according to the action, i.e. only relative performance determines relative success - that is, there is no discrimination by race, gender, age, etc. Suppose further that individuals have access to the same technology and have the same taste over the actions they take and rewards they receive. Suppose individual endowments do not

affect an individual’s ability to participate in the tournament. Suppose distributions of endowments and rewards are exogenous and independent of each other, and that aggregate actions do not change aggregate rewards and have no social value. Is a particular individual happier when endowments are more equal, or when rewards are more equal?

To answer this question, we first note that, for two types of inequality, the results can be opposite. However, the precise way how these two types of inequality affect a particular individual depends on the individual’s starting position. Moreover, it is greater (in)equality of *rewards*, not of *endowments*, that has the conventional properties of (in)equality. We summarize these as follows.

Remark 4 *For most individuals, change in the equality of endowments has qualitatively opposite effects to a change in the equality of rewards.*

(i) *Greater equality of endowments tends to increase effort for most and to make the middle and upper “classes” worse off.*

(ii) *Greater equality of rewards tends to decrease effort for most and make the lower and middle “classes” better off.*

Rank-Indexing: Fixed r						
	Endowments z more equal			Rewards s more equal		
	Bottom	Middle	Upper	Bottom	Middle	Upper
	$z_p > z_a$	$z_p \sim z_a$	$z_p < z_a$	$s_p > s_a$	$s_p \sim s_a$	$s_p < s_a$
Direct	$U_p > U_a$	$U_p \sim U_a$	$U_p < U_a$	$U_p > U_a$	$U_p \sim U_a$	$U_p < U_a$
Total	$U_p \gtrsim U_a$	$U_p < U_a$	$U_p < U_a$	$U_p > U_a$	$U_p > U_a$	$U_p \gtrsim U_a$
Level-Indexing: Fixed z						
	Endowments z more equal			Rewards s more equal		
	Bottom	Middle	Upper	Bottom	Middle	Upper
	$r_p < r_a$	$r_p \sim r_a$	$r_p > r_a$	$s_p > s_a$	$s_p \sim s_a$	$s_p < s_a$
Direct	$U_p < U_a$	$U_p \sim U_a$	$U_p > U_a$	$U_p > U_a$	$U_p \sim U_a$	$U_p < U_a$
Total	$U_p < U_a$	$U_p < U_a$	$U_p \lesssim U_a$	$U_p > U_a$	$U_p > U_a$	$U_p \gtrsim U_a$

Table 1: Summary of the effects on individual equilibrium utility by individual’s starting position in the distribution of endowments: direct vs. total effect.

The summary of the effects on equilibrium utility is presented in Table 1. Clearly, there is some difference whether we index by rank or by level. However, it is also clear that those in the middle of the human hierarchy will be happier with more equal rewards than with more equal endowments whatever the means of comparison.

7 Which Inequality?

In this paper, we focus mainly on two different types of inequality - inequality of endowments and inequality of rewards with our analysis done under the assumption of (competitive) equality of opportunity. In the context of a simple model of social competition, we are able to give formal, mathematical definitions of these three concepts.¹⁰ Approaching the issue of inequality from within economic theory, our terminology may not easily map into the terminology widely used by moral philosophers. In this section, we try to show how some aspects of the pre-existing concepts relate to our framework.

Our methodology is different from the one typically used by those interested in distributional justice for a number of reasons. First, much research on inequality assumes explicitly or implicitly that redistribution of resources can benefit some only at the cost of others. Thus, it is purely a question of subjective judgement, formalized in terms of a social welfare function, which of the many possible Pareto-efficient distributions of resources is socially optimal. In contrast, in the presence of relative concerns or inter-personal competition, the effects of redistribution are not so straightforward. Thus, second, we consider a world of imperfect competition and asymmetric information, which necessarily we must analyze using game-theoretic methods.¹¹ Thirdly, we focus on how distributional changes affect each individual's welfare, rather than on the welfare of the entire society. Finally, this work is descriptive in that we analyze who loses and who gains from changes in equality, without any attempt to evaluate normatively these gains and losses.

The possibility of spillovers from one individual's choices to other individuals' well-being seemed to have attracted little attentions from moral philosophers. While Nozick (1974) did briefly touch on this issue in his chapter "Equality, Envy, Exploitation, etc.", we are not aware of an existing theory of justice that allocates interpersonal competition a central role. However, we do not deny the importance of moral judgement in this area. Indeed, we hope that our work may provide a potential input into the work of moral philosophers. Here we try to sketch briefly how we think our work relates to the existing literature on distributive justice.

One of the early taxonomies of inequality is due to Rousseau (1754), who analyzed two types of inequality - *natural* ("established by nature") and *moral* ("established, or at least authorised by the consent of men").¹² It is this second, moral, inequality, that

¹⁰Formal economic modelling of inequality is not new - see Roemer (1996) for an excellent account of the formal economic approach to the theories of justice.

¹¹Despite the enormous impact of such ideas on many areas of economics, strangely they seem to have had little impact on the study of inequality.

¹²See Rousseau (1754, Part I): "I conceive that there are two kinds of inequality among the human species; one, which I call natural or physical, because it is established by nature, and consists in a difference of age, health, bodily strength, and the qualities of the mind or of the soul: and another, which may be called moral or political inequality, because it depends on a kind of convention, and is established, or at least authorised by the consent of men. This latter consists of the different privileges, which some men enjoy to the prejudice of others; such as that of being more rich, more honored, more

Rousseau criticized, thus establishing the long-running interest in inequalities created or authorized by society. Our taxonomy of inequality differs from the one established by Rousseau as, in his treatment of “moral” inequality, Rousseau does not distinguish between inequality of initial endowments and inequality of what one can obtain as the result of social arrangements - i.e. of what we call rewards. Further, we consider both “natural” endowments - such as abilities, talent, physical attractiveness, and so on - as well as endowments that have economic, social, or “moral”, origin - such as income, wealth, capital goods, and so on. However, we argue that whether endowments are natural or socially created is still important in determining which of two different analytical methods, indexing by level and indexing by rank, is more appropriate. We introduced the distinction between the two types of endowments in Section 3, and went on to show in Sections 4 and 5 (where we consider, respectively, the “moral” and “natural” sources of heterogeneity in endowments) that the two methods generate different perspectives on the effects of changes in inequality.

In the past few decades the debate on “equality of what” has been re-ignited by Sen (1980), who criticized the utilitarian approach, the total utility approach, and the approach of Rawls (1971), pointing out that individuals may differ in their ways of utilizing endowments (their “technology”), or in their tastes.¹³ Here, we assume that individuals differ from each other in their endowments only, so that their tastes and “technologies” they employ (that is, the way they transform their endowments into performance) are the same. While these assumptions are undoubtedly restrictive, they allow us to focus on the main question - that is, whether a particular individual’s welfare is higher under inequality of endowments or under inequality of rewards. The “expensive taste” argument, not dissimilar to the one presented by Sen (1980) was also put forward by Dworkin (1981a) against pursuit of equality of welfare. Instead, Dworkin (1981b) advocated for equality of resources. However, as in most of the existing literature on distributive justice, Dworkin (1981b) does not seem to distinguish the timing or function of different economic resources, which is crucial to our distinction between endowments and rewards.

The importance of inequality of *rewards* as incentive mechanism has been pointed out by sociologists Davis and Moore (1945). In their functional theory of stratification, they argued that a hierarchical system of rewards is necessary as it allocates talent and provides motivation to exert effort and/or acquire skills necessary for functionally important or ability-demanding jobs. Similarly, Rawls (1971) accepted that inequality of rewards can be justified only as a motivational tool. Our results support the motivational justification of inequality of rewards. Since more unequal rewards tend to increase the return to effort, effort tends to increase when rewards are more unequal. We also point out that while the inequality of rewards matters for incentives and thus in-

powerful or even in a position to exact obedience.”

¹³Sen (1980) put forward a radically different idea, which is difficult to capture with traditional economic modelling tools. He advocated for equality of what he called “basic capabilities” - such as abilities to move around, to meet nutritional requirements, to participate in the social life of the community and so on.

dividual welfare, so does the inequality of endowments. Moreover, as we show, for each individual, the incentive effects (and under some conditions, total effects) of the two types of inequality may be (qualitatively) the opposite of each other.

More recently, the attention of justice theorists has shifted towards equality of *opportunity* (see Rawls (1971) and Roemer (1996) among others). In practice, equal opportunity is said to exist when people with similar abilities reach similar results (equality of outcome). Importantly for our analysis is that equality of opportunity can take various forms. One form is focused on the *technologies* available to the individuals. For example, one can “level the playing field” and thus increase the rewards (or the chances to receive a particular reward) of the disadvantaged by either providing them with superior technology, or handicapping those better-endowed by making more difficult for them to exert effort. Another form, advocated by Roemer (1996, 1998), focuses on reward *schedules*, suggesting to equalize rewards to effort across individuals holding the same rank within the distribution of effort within their own type.

Alternatively, equality of opportunity may be affected through the *rules* according to which distribution of rewards is made. That is, rewards can be allocated to individuals not only on the basis of their performance (such as output or educational achievements) but also based on other characteristics (such as initial endowment, socio-economic status, gender, age, moral qualities, etc.). Lloyd Thomas (1977) pointed out that it is important to distinguish the equality of opportunity when allocation involves abundant rewards (e.g. everyone who wants to receive tertiary education will receive some form of it) from the one involving scarce rewards (e.g. everyone who wants to get a particular job will get it). He noted that improving some individuals’ opportunity of receiving scarce rewards may involve “compulsory reducing the opportunities others already have”. Instead, he advocated “fair competition for scarce resources” - an argument consistent with Nozick (1974)’s argument in support of fair rules. The argument in support of *competitive equality of opportunity* was further advanced by Green (1989), who argued that in the absence of perfect information about individual abilities, talents, or endowments, society may often need to elicit these (or at least to be able to rank individuals in the order of their endowments of productive resources), and competitions are common mechanisms for information revelation.

Here, we imagine a society where there is formal equality of opportunity. That is, “there is no legal bar to access to education, to all positions and jobs, and that all hiring is meritocratic” (Roemer, 1996, p. 163) and where individuals are rewarded according to their productivity (Lamont, 2003). We also assume that all individuals use the same technologies (that is, no individual is handicapped and no individual have an access to a superior technology). Further, the rewards to be distributed are scarce and the individual endowment of productive resources is each individual’s private information. Therefore, describing our tournament model as possessing competitive equality of opportunity has some justification.

Thus, in summary, our current work differs from prior work on distributive justice for two main reasons. First, existing work has not considered the externalities arising

from interpersonal competition and, therefore, has not taken a strategic approach to understanding the impact of inequality. Second, while our tournament model borrows the idea of equality of opportunity from the existing literature, to the best of our knowledge, ours is the first attempt to separate inequality in terms of its timing and function, between initial endowments and final rewards.

8 Discussion and Conclusions

This paper introduces a new distinction between different kinds of inequality. Equality of initial endowments and equality of the rewards to success in society have opposing effects. Greater equality of endowments increases the degree of social competition, greater equality of rewards reduces it. Thus, it is not the case that greater inequality necessarily decreases happiness. Rather, it is inequality of rewards, not of endowments, that is a likely cause of concern.

There has been much recent work concerned with the possibility that people have intrinsic preferences over the level of inequality. Here, we offer a reason why inequality may matter even without any concern for social justice and in the absence of such social preferences. This is because when there is interpersonal competition for employment and educational opportunities, inequality has a direct impact on incentives and, hence, equilibrium effort and equilibrium utility. The competitive threat of being excluded from desirable opportunities means that, in equilibrium, everyone works too hard. This means that people can be made better off by a change in incentives implicit in the two different forms of inequality. The majority can gain from a more dispersed distribution of endowments or from a less dispersed distribution of rewards. In fact, we can construct examples where a more equal distribution of rewards makes everyone better off, that is, it is Pareto improving, even though this reduction in incentives decreases total performance.

It is true that if competitors' efforts benefit other agents, such as partners, employers or members of future generations, then overall welfare judgements become more complicated. However, our analysis does point up that there can be unexpected losses from changes in the level of inequality. Gains to future generations may not be sufficient compensation to those who lose now from greater inequality of rewards. Or, as another possibility, societies with high inequality of rewards may have higher growth but lower happiness for a given level of income than societies with greater equality of rewards. Thus, one clear direction for further research is to use the current model as the stage game in a dynamic setting. Preliminary results in this direction indicate that the effects of changes in inequality on growth depend heavily on whether current performance determines the rewards or the endowments of the next generation.

As we demonstrated in this paper, the relationship between inequality and individual welfare can be less straightforward than is commonly thought. The gains and losses to greater equality are not uniform across society, and differ according to whether we

consider equality of endowments or of rewards. It even differs according to the viewpoint taken, that is, whether we compare at a constant level of endowment or at a constant rank in society. However, rather than being a setback, we believe the richness of the relationships we have outlined and the tools we have developed to analyze them offer many possibilities for greater understanding of social phenomena.

For example, one of the more recent reasons advanced for the desirability of greater income equality is the presence of relative concerns. It has been argued that in countries where gross poverty has been eliminated, health tends to be driven by stress caused by one's relative position, which, in turn, is exacerbated by inequalities. The most famous single case study is that of British civil servants, where health was found to be very strongly positively correlated with a civil servant's rank in the service (Marmot et al. (1991)). If this is the case, it has been argued by several authors, notably Frank (1999, 2000), that greater equality should be socially beneficial. However, we have seen in this paper that, even if utility does depend on relative position, it may not be the case that inequality has a negative impact on welfare. The fact that material outcomes depend on interpersonal competition may in fact lead to utility increasing with greater inequality. Indeed, the empirical evidence as a whole, surveyed in Deaton (2003), does not support a general link between inequality and ill health. Equally, it has been difficult to establish whether there is a positive or negative relationship between inequality and self-reported happiness or life-satisfaction (Alesina et al. (2004), Clark (2003)).

This paper suggests a reason why this may be the case. Even when utility depends on relative position, different types of inequality may have opposite effects. Therefore, empirical work that is based on measures of inequality that conflate rewards and endowments may obtain weak results as the two opposing effects may cancel. The problem in immediately applying this insight to empirical problems is that, to our knowledge, no distinction between reward and endowment inequality has traditionally been made in data collection. However, with data sources such as longitudinal studies becoming more widely available, it may soon be possible to distinguish between initial endowments and final rewards.

Finally, we would like to emphasize that the fact that this work approaches inequality outside the framework of distributive justice does not mean that moral considerations are irrelevant to the issue of inequality. In fact, precisely because existing theories of justice do not give interpersonal competition such a central role, our tournament model may provide new tools and new insights that may be useful to researchers on distributive justice. Thus, we hope that this paper, even though it takes a purely economic approach to the analysis of inequality, may aid the understanding of the non-economic aspects as well.

Appendix A: The Dispersive, Star and Other Stochastic Orders

We use two different stochastic orders, the dispersive and the star orders. These may not be well known in economics (though see Hoppe et al. (2005)), but are extremely useful for the social contests we consider. Let F and G be two arbitrary continuous distribution functions each with support on an interval (but the two intervals need not be identical or even overlap) and let F^{-1} and G^{-1} be the corresponding left-continuous inverses (so that $F^{-1}(r) = \inf\{x : F(x) \geq r\}$, $r \in [0, 1]$ and $G^{-1}(r) = \inf\{x : G(x) \geq r\}$, $r \in [0, 1]$), and let f and g be the respective densities.

Definition 1 (*Shaked and Shanthikumar (1994)*) *A variable with distribution F is said to be smaller in the dispersive order (or less dispersed) than a variable with a distribution G (denoted as $F \leq_d G$) whenever $G^{-1}(r) - F^{-1}(r)$ is (weakly) increasing for $r \in (0, 1)$.*

That is, the difference in the two variables at a given rank increases in rank. This has the following important consequence,

$$G \geq_d F \text{ if and only if } f(F^{-1}(r)) \geq g(G^{-1}(r)) \text{ for all } r \in (0, 1) \quad (18)$$

That is, for a fixed rank, the more dispersed distribution is less dense than the less dispersed one. Note that because the condition (18) is expressed in terms of ranks, there is no problem in comparing distributions with different, even disjoint, supports. Finally, when both distributions have finite means, if F is less dispersed than G then $\text{Var}_F(z) \leq \text{Var}_G(z)$ whenever $\text{Var}_G(z) < \infty$. Figure 7 shows a simple example of distributions which are ordered in terms of the dispersion order. The distributions G_B^1, G_B^2, G_B^3 all have different means but are equally dispersed and all are more dispersed than G_A . Figure 8 shows the importance of the dispersion order for incentives in the tournament model: if a distribution H_a is more dispersed than a distribution H_p then by (18) necessarily the inverse function $S_a(r)$ is steeper than $S_p(r)$. This is because if $S(r) = H^{-1}(r)$, then $S'(r) = 1/h(H^{-1}(r))$.

The star order is defined as follows.

Definition 2 (*Shaked and Shanthikumar (1994, p105)*). *A variable with a distribution G is larger than a variable with a distribution F , or $G \geq_* F$, in the star order if $G^{-1}(F(z))/z$ increases for $z \geq 0$.*

Note that if X and Y are two non-negative random variables then

$$X \leq_* Y \iff \log X \leq_d \log Y \quad (19)$$

But unfortunately if a distribution F is more dispersed than another distribution G , or $F \geq_d G$, it does not imply that $F \geq_* G$, though it is not excluded. Nor does $F \geq_* G$ imply $F \geq_d G$, nor does it rule it out.

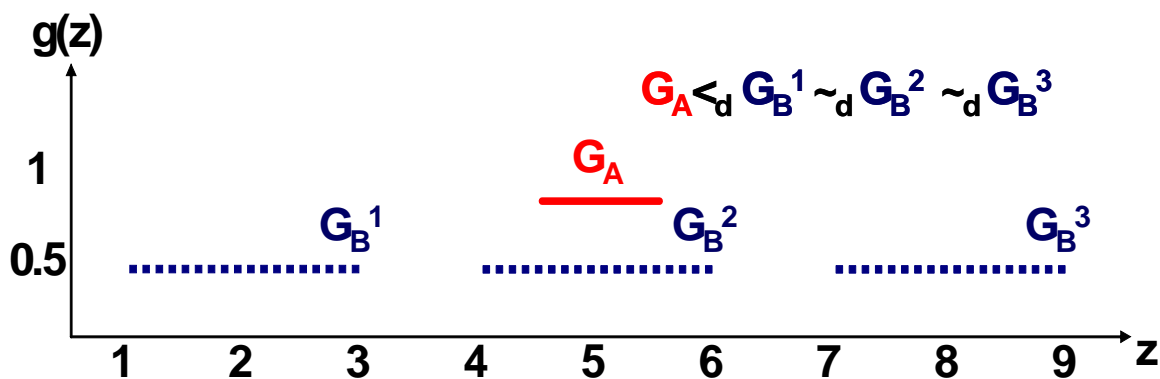


Figure 7: An example of the dispersion order.

Lemma 1 Take two distributions $H_a(s)$, $H_p(s)$ with support on the positive real line and with differentiable inverses $S_a(r)$ and $S_p(r)$ respectively. Then, the following holds

$$H_p(s) \geq_* H_a(s) \Leftrightarrow \frac{d S_p(r)}{dr S_a(r)} \geq 0 \Leftrightarrow \frac{S'_p(r)}{S_p(r)} \geq \frac{S'_a(r)}{S_a(r)} \quad (20)$$

for all $r \in (0, 1)$.

Proof: The relationship between the first and second statements follows directly from Shaked and Shanthikumar (1994, pp70-71 and Theorem 3.C.1). The relation between the second and third follows from differentiation. ■

Economists often use second order stochastic dominance to order distributions in terms of dispersion, there is no clear relation between the dispersive order and second order stochastic dominance. This is because, in everyday terms, one distribution can second order stochastically dominates another if it is either higher or less dispersed, while the dispersive order is only concerned with dispersion. Note also that if $H_a \geq_d H_p$, the distribution H_a is more dispersed but, for example, they have the same means, it may well be true that distribution H_p second order stochastically dominates H_a . The star order is much closer to second order stochastic dominance in that if distribution H_a is larger in the star order $H_a \geq_* H_p$ than H_p , then it is larger in the Lorenz order (Shaked and Shanthikumar, (1994, p107), which is equivalent to second order stochastic dominance if the two distributions have the same mean.¹⁴ However, one says that the less dispersed distribution second order stochastically dominates the more dispersed, which is the other way round from the star and dispersive order where the if a distribution is “larger” then typically it is more dispersed. See the following examples.

Example 2 If $H_a(s) = s$, that it is uniform on $[0, 1]$ and $H_p(s) = 2s - 1/2$, a uniform distribution on $[1/4, 3/4]$, then in many ways H_a is more dispersed than H_p . Indeed,

¹⁴Second order stochastic dominance is therefore sometimes referred to as the generalized Lorenz order.

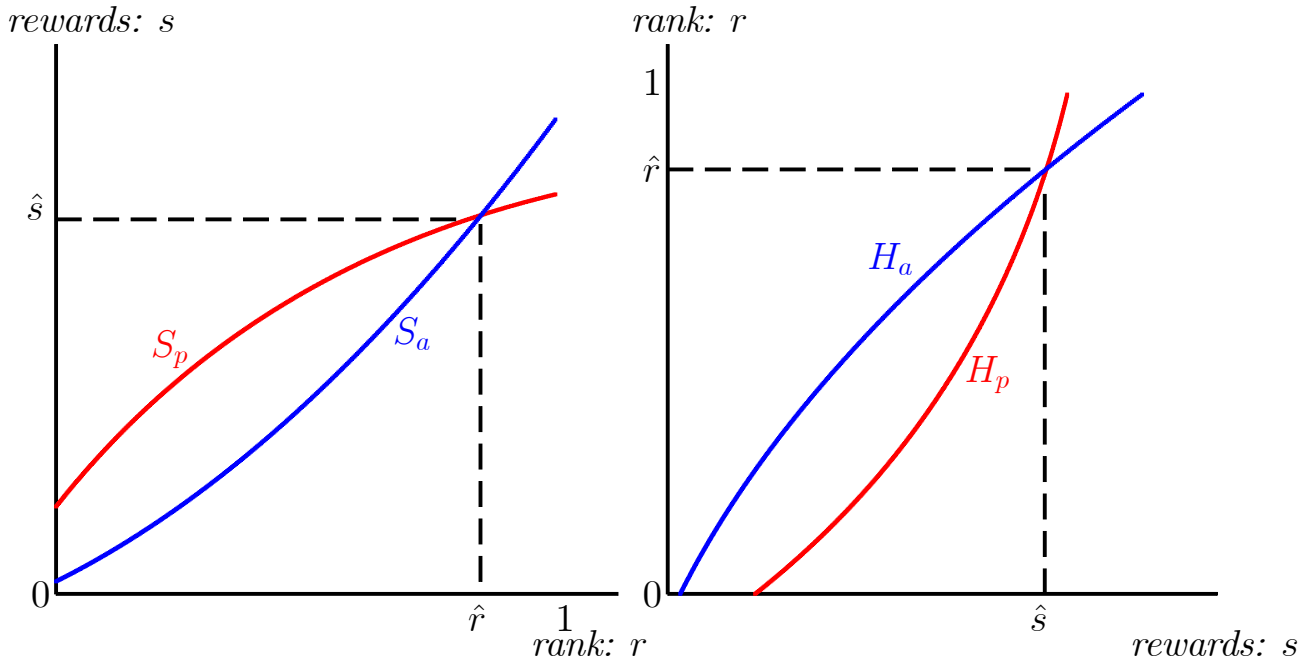


Figure 8: Dispersion order: If the ex post distribution is less dispersed than the ex ante, or $H_p \leq H_a$ then the inverse distribution function $S_p = H_p^{-1}(r)$ is less steep than S_a for all $r \in (0, 1)$, i.e. the marginal return to an increase in rank is lower.

$S_a(r)/S_p(r) = r/(r/2 + 1)$ which is increasing so $H_a \geq_* H_p$. Furthermore, $S'_a(r) = 1 > 1/2 = S'_p(r)$ so that $H_a \geq_d H_p$. And finally H_p second order stochastically dominates H_a .

This example illustrates a more substantive difference.

Example 3 If $H_a(s) = s - 2$, that it is uniform on $[2, 3]$ and $H_p(s) = (s - 1)/2$, a uniform distribution on $[1, 3]$, then H_p is more dispersed than H_a but stochastically lower. The dispersive order captures the dispersion so as $S'_a(r) = 1 < 2 = S'_p(r)$ so that $H_p \geq_d H_a$. But, $S_p(r)/S_a(r) = (2r + 1)/(2 + r)$ which is increasing so $H_p \geq_* H_a$. However, as H_a stochastically dominates H_p , it also second order stochastically dominates H_p .

Appendix B: Proofs

Proof of Proposition 1: Mailath (1987) establishes in a general signaling model the existence and uniqueness of a separating equilibrium under certain conditions. If the current model fits within Mailath's framework, then it would follow that the unique separating equilibrium is a solution to the differential equation (5) with boundary condition $x(0) = x_c(0)$ from Theorems 1 and 2 of Mailath (1987, p1353). It would also

follow by Proposition 3 of Mailath (1987, p1362) that $x(z) > x_c(z)$ on (\underline{z}, \bar{z}) . The only substantial difference is that Mailath assumes the signaller's utility is of the form (in current notation) $V(r, \hat{r}, x)$ where V is a smooth utility function and \hat{r} is the perceived type, so that in a separating equilibrium the signaler has utility $V(r, r, x)$. To apply this here, first, fix $G(z)$ and $H(s)$. Now, clearly, one can define the function $V(\cdot)$ such that $V(r, \hat{r}, x) = U(x, Z(r) - x, S(\hat{r}))$ everywhere on $[0, 1] \times [0, 1] \times [\underline{z}, \bar{z}]$. One can then verify that the conditions (i)-(v) imposed on $U(\cdot)$ imply conditions (1)-(5) of Mailath (1987, p1352) on V .¹⁵ In particular, note that condition (1) is simply that V is twice differentiable, condition (2) is that $V_2 \neq 0$, here $V_2 = U_s S'(r) > 0$. Condition (3) is that $V_{13} \neq 0$ and here $V_{13} = (U_{xy} - U_{yy})Z'(r) > 0$. Mailath's condition (4) requires that $V_3(r, r, x) = 0$ has a unique solution in x which maximizes $V(r, r, x)$. Here, $V_3 = U_x - U_y$ and we have assumed under condition (v) that there is a unique solution to the equation $U_x - U_y = 0$. Since here $V_{33} = U_{xx} - 2U_{xy} + U_{yy} < 0$, this solution is a maximum. Furthermore, since V_{33} is everywhere negative, Mailath's condition (5) is automatically satisfied. ■

Proof of Proposition 2: First note that, given the equation (5), we have that

$$\frac{x'_a(r)}{x'_p(r)} = \frac{\phi(Z_a(r), S(r), x_a)}{\phi(Z_p(r), S(r), x_p)} \quad (21)$$

so that any point where $x_a = x_p$ the relative slope only depends on Z_a and Z_p , and thus the slopes are equal whenever Z_a and Z_p are equal. Furthermore, given our assumptions, we have that

$$\frac{\partial \phi(z, s, x)}{\partial z} = \frac{U_{ys}(U_y - U_x) - U_s(U_{yy} - U_{xy})}{(U_y - U_x)^2} > 0 \quad (22)$$

(by properties (iii) and (iv), it holds that $U_y - U_x > 0$ when evaluated at the equilibrium solution as $x(r) > x_c(r)$). Thus, at any point where $x_a(r) = x_p(r)$ we have that $x'_a > x'_p$ (so that x_a is steeper than x_p and thus crosses x_p from below) whenever $Z_a(r) > Z_p(r)$ (i.e. whenever ex-ante endowments exceed ex-post endowments), and vice versa.

By the boundary conditions (7), the condition $Z_a(0) \leq Z_p(0)$ implies that $x_p(0) \geq x_a(0)$ (i.e. that the poorest individual, now that she has greater endowments chooses greater performance). Given our assumption that G_a and G_p are distinct it follows that $Z_p(r) > Z_a(r)$ almost everywhere on $(0, \hat{r}]$. Thus, $x_p(r)$ can only cross $x_a(r)$ from below except perhaps at the finite number of points where $Z_p(r) = Z_a(r)$.

We first rule out that there is an interval where $x_p(r) \leq x_a(r)$. Suppose on the contrary there exist at least one interval $[r_1, r_2] \subseteq [0, \hat{r}]$ such that $x_p(r) \leq x_a(r)$. By the continuity of x_a and x_p , it must be that $x_p(r_1) = x_a(r_1)$. Note that

$$\frac{\partial \phi(z, s, x)}{\partial x} = \frac{(U_{xs} - U_{ys})(U_y - U_x) - U_s(2U_{xy} - U_{xx} - U_{yy})}{(U_y - U_x)^2} < 0. \quad (23)$$

¹⁵Mailath, in proving the intermediate result Proposition 5 (1987, p1364), also assumes that $\partial V/\partial \hat{r}$ is bounded. Here, if we assume that both U_s and $S'(r)$ are bounded (the latter requires $h(s)$ is non-zero on its support), this result will also hold.

In combination with (22), it would follow that $x'_a(r) < x'_p(r)$ almost everywhere on $[r_1, r_2]$, which combined with $x_a(r_1) = x_p(r_1)$ is a contradiction to $x_p(r) \leq x_a(r)$ on the interval. Thus, $x_p(r) > x_a(r)$ almost everywhere on $[0, \hat{r}]$.

We next rule out that $x_p(r) = x_a(r)$ at individual points. By the previous argument that excludes intervals where $x_p(r) \leq x_a(r)$, this is only possible at the isolated points where $Z_p(r) = Z_a(r)$. But at any such point \tilde{r} on $(0, \hat{r}]$, as $Z_p(r) > Z_a(r)$ almost everywhere, we have that $g_p(Z_p(\tilde{r})) \geq g_a(Z_a(\tilde{r}))$ (remember that $Z'(r) = 1/g(Z(r))$). Now, note that $Z_p(\tilde{r}) = Z_a(\tilde{r}) = \tilde{z}$. Next, we invoke the level-indexing approach and consider solutions to the game in terms of endowments z . Let $S(z) = H^{-1}(G(z))$. Write solutions to the differential equation (17) as $x_p(z)$ and $x_a(z)$ for the respective distributions of endowments. Then if $x_p(\tilde{r}) = x_a(\tilde{r})$, it must be that $x_p(\tilde{z}) = x_a(\tilde{z})$. As $x_p(r) > x_a(r)$ for r in $(\tilde{r} - \epsilon, \tilde{r})$ for some $\epsilon > 0$, we must have $x_p(z) > x_a(z)$ for endowments slightly less than \tilde{z} . Note that it must hold that $x'_p(\tilde{r}) = x'_a(\tilde{r})$, and for the case of $g_p(\tilde{z}) > g_a(\tilde{z})$, it must be that $x'_p(\tilde{z}) > x'_a(\tilde{z})$ so that $x_p(z)$ crosses $x_a(z)$ from below, which is a contradiction. This leaves us with the possibility that $x_p(r) = x_a(r)$ in a non-generic case of $g_p(Z_p(\tilde{r})) = g_a(Z_a(\tilde{r}))$. ■

Proof of Proposition 3: First, as endowments are (weakly) higher at $r = 0$, the privately optimal performance will be higher ex post $x_{c,p}(0) \geq x_{c,a}(0)$ as will equilibrium performance at $r = 0$ by the boundary conditions (7). Thus, $U_p(0) \geq U_a(0)$ (i.e. as the poorest individual has no reduction in endowments she will not be worse off). We have that

$$\frac{1}{g_p(Z_p(r))} = \frac{dZ_p(r)}{dr} \geq \frac{dZ_a(r)}{dr} = \frac{1}{g_a(Z(r))} \text{ for all } r \in [0, 1]$$

In other words, $Z_p(r)$ is (weakly) steeper than $Z_a(r)$ on $[0, 1]$, so that clearly $Z_p(r) \geq Z_a(r)$ for $r \in [0, 1]$.

Suppose that $U_p(0) > U_a(0)$, and suppose, in contradiction to the claim we are trying to prove, that $U_p(r)$ equals $U_a(r)$ at least once on $(0, 1)$. Denote the first such point as $r_1 \in (0, 1)$. It is easy to show that, as $Z_p(0) \geq Z_a(0)$ and $G_p \geq_d G_a$, we have $Z_p(r) > Z_a(r)$ for all $r \in (0, 1]$. Thus, by Corollary 1, $x_p(r) > x_a(r)$ on $(0, 1]$, and it must be that $y_p(r) < y_a(r)$ in the neighborhood of r_1 . Let $U_{i,y}(r) = U_y(x_i(r), Z_i(r) - x_i(r), S(r))$ for $i = a, p$. Then, as $dU_y = U_{xy}dx + U_{yy}dy$, and, given our original assumptions on U , it must be that $U_{p,y}(r) > U_{a,y}(r)$ in a neighborhood of r_1 . Using the marginal utility condition (8), combined with the fact that, given the dispersion order, $g(Z_p(r)) \leq g(Z_a(r))$, it must be that $U'_p(r) > U'_a(r)$ in a neighborhood of r_1 , so that $U_p(r)$ can only be steeper than $U_a(r)$, and thus can only cross from below. Given $U_p(0) > U_a(0)$, we are done.

If instead we have that $U_p(0) = U_a(0)$, then, by the above argument which rules out that U_p can cross U_a from above, the claim can only fail if there is an interval $(0, \tilde{r})$ on which $U_p(r) \leq U_a(r)$. Then, there must exist a point $r_2 \in (0, \tilde{r})$ such that $U'_p(r_2) \leq U'_a(r_2)$ and $U_{p,y} \leq U_{a,y}$. But given (8) and that $G_p \geq_d G_a$, if $U'_p(r_2) \leq U'_a(r_2)$ then $U_{p,y}(r_2) \leq U_{a,y}(r_2)$, which can only happen if $y_p(r_2) \geq y_a(r_2)$. But this, combined with the fact that $x_p(r_2) > x_a(r_2)$ (by Proposition 2) implies that $U_p(r_2) > U_a(r_2)$,

which is a contradiction. ■

Proof of Proposition 4: From Proposition 2, we have $x_p(r) > x_a(r)$ on $(0, \hat{r}]$. But note as here $Z_p(0) > Z_a(0)$, the lowest agent has a strictly greater endowment, we have also $x_p(0) > x_a(0)$ as the cooperative choice, which is the equilibrium choice of the bottom agent by (7), is increasing in endowments. Turning to utility, we can consider two cases. First, suppose that $x_p(r) \geq x_a(r)$ on $[\hat{r}, 1]$. Then, as endowments for individuals with rank $(\hat{r}, 1]$ are strictly lower ex-post than ex-ante, we have necessarily $y_p(r) < y_a(r)$ on $[\hat{r}, 1]$. Now, as $x_p(r) \geq x_a(r)$ and $y_p(r) < y_a(r)$, we then for some \tilde{r} can find a pair (\tilde{x}, \tilde{y}) such that $\tilde{x} + \tilde{y} = x_p + y_p$ (that is, (\tilde{x}, \tilde{y}) are feasible given ex-post endowments) but $x_{c,p} < \tilde{x} < x_p$ and $\tilde{y} = y_a$. But then, $U(x_p(r), y_p(r), S(r)) < U(\tilde{x}, \tilde{y}, S(r)) < U(x_a(r), y_a(r), S(r))$, and the result follows.

Suppose now instead that $x_p(r) < x_a(r)$ for some r in (r_1, r_2) with $r_1 > \hat{r}$. If $y_p(r) \leq y_a(r)$ on that interval, it is clear that $U_p(r) < U_a(r)$ and we are done. Suppose instead that $y_p(r) > y_a(r)$ on some interval (r_3, r_4) with $r_4 \leq r_2$ (as endowments are lower ex post for $r > \hat{r}$, it must be that $r_3 > r_1$). We want to rule out the possibility of $U_p(r) \geq U_a(r)$ somewhere on this interval. Now, it must be the case that $U_p(r_3) < U_a(r_3)$ as $x_p(r_3) < x_a(r_3)$ and $y_p(r_3) = y_a(r_3)$. We have $g_p(r) \geq g_a(r)$ everywhere. Furthermore, $dU_y = U_{xy}dx + U_{yy}dy$. Given that x decreases and y increases ex post on (r_3, r_4) and our original assumptions on U , it can be calculated that, given (8), that $U'_p(r) < U'_a(r)$ on this interval. Combined with $U_p(r_3) < U_a(r_3)$, the result follows. ■

Proof of Proposition 5: First, given the boundary condition (7), we have $x(0) = x_c(0)$. Note that applying property (v) to the definition of $x_c(r)$ in (6), we have $\partial x_c / \partial s \leq 0$ so that given $S_p(0) < S_a(0)$, it follows that $x_p(0) \geq x_a(0)$. Almost everywhere on $[0, \hat{r})$, we have both $S_a(r) > S_p(r)$ and $S'_p(r) > S'_a(r)$. Note that

$$\frac{\partial \phi(z, s, x)}{\partial s} = \frac{U_{ss}(U_y - U_x) - U_s(U_{ys} - U_{xs})}{(U_y - U_x)^2} \leq 0. \quad (24)$$

It immediately follows that if $x_a(r) = x_p(r)$ anywhere on $[0, \hat{r})$, $x'_a(r) > x'_p(r)$. So, there can only be one crossing of $x_a(r)$ and $x_p(r)$ on that interval and $x_p(r)$ must cut $x_a(r)$ from below. Thus, the only way for the claim to be false is if $x_p(r) \leq x_a(r)$ on some interval $[0, r_1]$. But then, as $\partial \phi(z, s, x) / \partial x < 0$ by (23) and $\partial \phi(z, s, x) / \partial s \leq 0$ by (24), and as $S_p(r) < S_a(r)$ and $S'_p(r) > S'_a(r)$, it follows that $x'_p(r) > x'_a(r)$ on $[0, r_1]$, which is a contradiction. ■

Proof of Proposition 6: Given additively separable utility, we have $x_p(0) = x_a(0) = x_c(0)$ as with separable utility the cooperative choice does not depend on $S(0)$. The differential equation (5) is now

$$x'(r) = \frac{S'(r)}{V_y(x, Z(r) - x) - V_x(x, Z(r) - x)} \quad (25)$$

Given the dispersion order, we have $S'_p(r) \geq S'_a(r)$ for all r and the result is easy to establish using the arguments in the proof of the previous proposition. ■

Proof of Proposition 7: As with additive separable utility, we have $x_p(0) = x_a(0)$ irrespective of $S_a(0)$ or $S_p(0)$. The differential equation is now

$$x'(r) = \frac{S'(r)}{S(r)} \frac{V(x, Z(r) - x)}{V_y(x, Z(r) - x) - V_x(x, Z(r) - x)}.$$

Now, by Lemma 1 in Appendix A, by the star order we have $S'_p(r)/S_p(r) \geq S'_a(r)/S_a(r)$ for all r . The proof again then follows that of Proposition 5. ■

Proof of Proposition 8: Given the lowest reward $S(0)$ is higher ex post, we have $U_p(0) > U_a(0)$. We divide $[0, 1]$ into two sets. Let I_1 consist of points where $x_p(r) \geq x_a(r)$ and I_2 consist of points where $x_p(r) < x_a(r)$. Considering I_2 , as rewards are higher and effort lower, clearly $U_p(r) > U_a(r)$ on I_2 . Turning to I_1 , here $x_p(r) \geq x_a(r)$ and hence $y_p(r) \leq y_a(r)$. Now, as $U'(r) = U_y S(r)/g(Z(r))$ and $dU_y = U_{xy} dx + U_{yy} dy$, we have $U'_p(r) > U'_a(r)$ almost everywhere on I_1 . The result follows. ■

Proof of Proposition 9: We have $S_a(r) < S_p(r)$ and $S'_p(r) < S'_a(r)$ on $[0, \hat{r}]$. Thus, by reversing Proposition 5, we have $x_a(r) > x_p(r)$ on $(0, \hat{r}]$. Furthermore, given that \hat{r} is the first point of crossing, we have $S_a(r) < S_p(r)$ on $[0, \hat{r}]$. It is clear that, as performance is strictly lower and rewards are higher under distribution $H_p(s)$, it follows that $U_p(r) > U_a(r)$. ■

Proof of Proposition 10: We have by the envelope theorem $U'(z) = U_y(x(z), z - x(z), S(z))$. First, we look at (i). Suppose the claim is false, and there exists at least one interval on $(\underline{z}, \hat{z}]$ where $U_p(z) > U_a(z)$. Let us denote the set of points as $I_U = \{z \leq \hat{z} : U_p(z) > U_a(z)\}$ (possibly disjoint), and let $z_1 = \inf I_U \geq \underline{z}$. We can find a $z_2 \in I_U$ such that $U_p(z) > U_a(z)$ for all z in $(z_1, z_2]$. Note that since, by the common boundary condition, $U_p(\underline{z}) = U_a(\underline{z})$. As $G_p(z) \leq G_a(z)$, then $S_p(z) \leq S_a(z)$ for all $z \in I_U$. As rewards are lower, for $U_p(z) > U_a(z)$ to be possible, it must be the case that $x_A(z) < x_B(z)$ for all $z \in I_U$. But then as U' is increasing in $x(z)$ and strictly increasing in $S(z)$, we have $U'_p(z) \leq U'_a(z)$ on I_U . This, together with $U_p(z_1) = U_a(z_1)$, implies $U_p(z) \leq U_a(z)$ for all $z \in (z_1, z_2]$, which is a contradiction. Part (ii) can be established by an identical argument ■

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