On the Faustian Dynamics of Policy and Political Power

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Abstract

This paper examines the "Faustian dynamics" of policy and power. We posit a general class of dynamic games in which the current ruler must choose a policy that may alter the future balance of power. The policies chosen by a political leader at date t (e.g., immigration, taxes, or public goods) alter the distribution of citizen characteristics (e.g., ethnicity, ideology, or wealth) in date $t+1$. Changes in the distribution of citizens' characteristics change the balance of political power in $t+1$ which, in turn, determines the policy choices at that date.

In a *Policy-endogenous (PE) equilibrium*, a transfer of power can result from one's endogenous policy choices. The current ruler cannot un-couple the direct effect of his policy from its indirect effect on future power. He must therefore craft a "Faustian bargain" from the following trade-off: if he chooses his preferred policy, he then sacrifices future political power; yet if he wants to preserve his future power, he must sacrifice his present policy objectives. We decompose this trade-off into two basic rationales. The *political preservation effect* induces the authority to choose "more conservatively" than if his policy choice did not affect his political fortunes. However, the *reformation effect* induces "more progressive" policies in order to exploit the productivity gains from policies chosen by more progressive successors. The general model characterizes equilibria with *monotone Faustian dynamics*: political power evolves toward more progressive types of leaders. In a parametric special case, we also characterize *cyclical Faustian dynamics*: political power oscillates between progressive and conservative types of leaders.

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1 Introduction

In Goethe's *Faust*, a well meaning Faust seeks knowledge, truth, and beauty, but cannot achieve them on his own. The devil appears and strikes a bargain with Faust: the devil will serve Faust while Faust remains here on earth; in exchange Faust must serve the devil in hell. But as part of the agreement, if Faust is so happy with the devil's services that he wants to "freeze" the present moment forever, then Faust must die immediately. Hence, the bargain endows Faust with the power to reach his objective but denies him the ability to savor it.[1]

So it is, quite often, with "political bargains." For if a political leader chooses a desirable but unpopular policy, he may lose power and thus lose the ability to shape policy in the future. By opting to keep political power, he must sacrifice his policy objectives and then face the same trade-off in the future. Hence, the political bargain endows a leader with the power to determine policy only as long as he does not choose the one he most prefers.

Historical examples of actors who face some sort of "Faustian trade-off" are common. Schonhardt-Bailey (2002, 2006) documents the political sacrifice made by Conservatives in Britain in the early 19th century:

> "In May of 1846, a British Parliament consisting predominantly of landowners decided to forego protection for agriculture by repealing the famous Corn Laws, a decision that split the Conservative party for a generation... Within a month of gaining repeal, the Peel Government fell and the Conservatives remained divided and for the most part out of office for decades to come." (Schonhardt-Bailey (2002, p.2))

Rauch (2006) argues that a different sort of Faustian bargain was struck by modern conservatives in the U.S. who pushed for tax cuts without cuts in government spending, concluding:

> "The conservative movement is in no position to accept or even acknowledge those implications [of a recent study showing tax cuts are associated with higher future spending], now that tax cutting has become the long pole in the Republican tent. Therein lies the element of tragedy. By turning a limited-government movement into an anti-tax movement, conservatism has effectively gone into business with the Big Government that it claims to oppose." (Rauch (2006))

This paper examines the nature of this "Faustian trade-off" in a dynamic game theoretic model in which policy and power are co-determinate. To highlight the "tragic" nature of these

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[1] While are there many versions of the famous Faustian Bargain, the most well known is rendered by Goethe in (translated) *Faust: The Tragedy (Part One and Part Two).* For English translation, see Bayard Taylor (exact publication date is unknown), in www.openlibrary.org/details/faustgoethe00goetiala .

kinds of choices, we do not presume that rulers crave power for its own sake. Instead, political actors are assumed to be purely policy-motivated. Their concern about losing power arises only because the new decision makers have different policy objectives than their own.

We are particularly interested in dynamic consequences of these Faustian bargains. Rather than elaborate on the specifics of one or another particular political mechanism, we therefore adopt the following "detail free" model of political aggregation. An ongoing society is inhabited by a continuum of infinitely lived citizens. At each date t, one of the citizens, whom we refer to as the current *leader* has effective authority to choose a policy that affects all the citizens in society. The "leader" can be viewed as an elected ruler, or alternatively, as a pivotal voter whose preferences are decisive in determining a policy. The key element here is that the policy choice of the leader in date t can change political power in a way that ultimately changes the identity of the leader in date $t + 1$. In other words, future political power is endogenously driven by current policy change. We refer to this as a case of *policy-endogenous (PE) political power*. Under policy-endogenous power, the dynamics induce a feedback loop from policy to power and back to policy.

In the "detail free" model, the policy-power link is specified in reduced form as a map from population characteristics to the type or identity of the leader. Behind the reduced form, we have in mind an electoral procedure. Current policies then affect future electoral outcomes by changing the underlying demographic and/or distributional characteristics of the population. For instance, an increase in a country's income tax changes the future distribution of income. In turn, this may bring about changes in future political power.[2] A leader faces Faustian trade-off when his preferred policy strips him of power and then places it in the hands of a less desirable ruler.

Critically, the current leader is unable to un-couple the direct effect of his policy from its indirect effect on the identity of a future decision maker. Consequently, a change in *de facto* power may occur despite no change in the formal (or *de jure*) rules of the political system. By contrast, Acemoglu and Robinson (2006) examine an economic mechanism through which the elites can preserve their de facto power despite the democratizing (de jure) changes in institutions. Their mechanism is more appropriate than ours when current elites have the flexibility to isolate or reverse the consequences of their policy choices; ours is more appropriate when this flexibility is lacking.[3]

While an abundant literature in political economy studies the link from political power to policy, less is known about the "reverse causal link," i.e., from policy to power. One reason for this is that much of the literature combines standard majority voting with order-preserving policy changes in income distribution. For instance, in political economy models of growth such as Bertola (1993), Alesina and Rodrik (1994), Krusell et al (1996, 1997) and Krusell and

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[2] Section 2 discusses the foundations for this type of connection in more detail.

[3] See Section 5 for an expanded discussion.

Rios-Rull (1999), taxation and capital accumulation is determined under majority voting.[4] But tax policy in these models does not change the relative wealth distribution because an increase in a country's income tax preserves the order statistics of the population through time. An individual who is richer than another today, is still expected to be richer after the tax increase goes into effect. Consequently, a fixed median voter emerges in equilibrium. But this means that there is no change in political power - at least not through this channel.

There are some recent studies in which policy-endogenous changes in power do occur in equilibrium. These include Hassler, et. al. (2003), Ortega (2005), Hassler, et. al. (2005), Azzimonti (2005), and Campante (2007). Each of these explore a particular bias, either in the political mechanism, or in the transition technology, for generating policy-endogenous change. There are still other studies that admit policy-endogenous change, but the effects are largely undone by institutional choices which are explicitly uncoupled from the policy itself. These include Acemoglu and Robinson (2000, 2001, 2006, 2007), Cervelatti, et. al. (2006), Jack and Lagunoff (2006a,b), and Lagunoff (2006a) We discuss these and other related works in more detail in the concluding section (Section 5).

The emphasis in the literature on "unbiased" political systems is a natural starting point. Yet, biased polities have, historically, been the rule rather than the exception. Until the late 19th century, most governments explicitly weighted votes by some form of wealth or property value. In democracies today the bias is less formulaic but no less real. In the U.S., for example, Senate representation is biased in favor of less populous states, hence toward characteristics of rural rather than urban voters. In addition, campaign contributions and the large expense of running modern political campaigns bias the outcomes in favor of wealthier citizens. In parliamentary systems, small minority parties and the voters they represent often have disproportionate influence in the formation of majority governments.

More importantly, biased systems produce linkages between current policy and future power. To study their effects, we develop our model in two stages. We first posit a parametric model of public investment under a biased political institution. We then extend the results to a general (non-parametric) model. In each case we characterize the *Policy-endogenous (PE) equilibria* — smooth Markov Perfect equilibria in policy-endogenous regimes. In the PE equilibrium, it is not so much a question of *whether* one loses power, but rather *to whom.* As a benchmark, PE equilibrium paths of policy and power are compared to those resulting from *permanent authority (PA) equilibria* — equilibria under which political power is permanent.[5]

In the parametric model, when investment is below its equilibrium steady state level, each leader chooses a lower level of government investment than he would if his authority

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[4] For a detailed review and comparison of literature in this tradition, see Krusell et al (1997) and Persson and Tabellini (2003).

[5] The focus on smooth Markov Perfect equilibria is natural for purposes of comparison because the policy rule is inherently Markov when authority is permanent. Results in Judd (2004) also suggest that smoothness is a natural selection device when multiple equilibria exist.

were permanent. Likewise, starting above the steady state investment, a leader chooses a larger investment than if his power were permanent. In either case, the Faustian trade-off produces a natural "conservativism" in the traditional sense that a leader's policy decisions are moderated. The consequence is that growth and political change become more gradual than they would be under permanent power.

Yet, despite the moderating effect on individual incentives, political power does evolve over time. We distinguish between two cases: monotone and cyclical dynamics. Under monotone dynamics, if, for instance, the initial state starts below the equilibrium steady state, then power moves progressively toward more fiscally liberal types — those who prefer larger increases in government spending. The political bias toward liberal types reinforces the natural pattern of capital accumulation. Under cyclical dynamics, power oscillates between liberal and conservative types, with the oscillations dampening over time. Here the bias countervails the pattern of capital accumulation — increases in public capital move the population distribution toward more fiscally conservative leaders.

The monotone results are generalized in the abstract model when certain supermodularity restrictions hold. Our main results characterize Policy-endogenous equilibria in terms of a "distortion-adjusted" Euler equation in which a leader's motives may be decomposed into two basic rationales. First, the "political preservation effect" induces the leader to choose "more conservatively" or "less progressively" than if his hold on power did not depend on his policy choice. This means that he chooses a policy that slows the rate political change, e.g., chooses a lower level of public investment if the state is below its steady state level, than if his power were permanent.

However, this preservation effect is moderated by a second rationale, the "reformation effect," which induces less conservative policy. The reformation effect comes from the complementarity between current and future policies. More progressive policies put future authority in the hands of even more progressive types of leaders thus changing the marginal productivity of policy decisions in the present.

We show that the preservation effect increases in magnitude through time. This drives any initial leader to be more conservative than his successors. Political power evolves toward more progressive leaders, and, in fact, the policy trajectory may be more progressive than under permanent authority in the long run.

The general model is introduced in Section 2. Section 3 elaborates on the parametric example. The example displays some of the salient features of political systems that gives rise to policy-endogenous power. Section 4 returns to the abstract model and contains the main decomposition result. Section 5 reviews the related literature and discusses various extensions of the present model. Section 6 is an Appendix with the proofs of all the results.

2 The General Model

2.1 The Environment

Society is comprised of a continuum $I = [0, 1]$ of infinitely lived *citizen-types.* The payoff to each citizen-type in each period $t = 0, 1, \ldots$ depends on a state variable and a policy decision. The distribution of citizens across these citizen types varies with the state through time. Let $\omega_t \in \Omega$ denote the state and $a_t \in A$ the policy choice. Assume both A and Ω are subsets of a finite dimensional space $\mathbb{R}^\ell$, with $A = [\underline{a}, \bar{a}]^\ell$, and $\Omega = [\underline{\omega}, \infty)^\ell \subset \mathbb{R}^\ell_+$.[6]

Given any sequence of states $\{\omega_t\}$ and policies $\{a_t\}$, the dynamic payoff to citizen-type $i \in I$ is

$$\sum_{t=0}^{\infty} \delta^t u(i, \omega_t, a_t) \tag{1}$$

where δ is a common discount factor, and the payoff function u is smooth, bounded from above, increasing in ω_t, and strictly concave and decreasing in a.

A Markov process determines the distribution over future states as a function of current states and actions. In the ensuing analysis, the process is assumed to be deterministic. Nothing substantive in our results so far changes if stochastic shocks are added.[7] Formally, let $\omega_{t+1} = Q(\omega_t, a_t)$. The transition function Q is assumed nonnegative, smooth with bounded first derivative, increasing in ω, strictly increasing in a_t, and (weakly) concave in both a_t and ω_t.

The assumptions on u and Q are intended to reflect the idea that the policy a_t is a collection of taxes or public investments that, while costly in the present, augment the future value of the state. In turn, the state ω_t is a collection of capital stocks or assets. An obvious example is an income tax that generates revenue to fund public infrastructure.

2.2 Permanent Authority: A Benchmark

Before proceeding with a model in which decision makers face Faustian trade-offs, we consider a case in which they do not. That is, some individual, labeled i_0, maintains political power regardless of his policy action. This "king" or "dictator" chooses a policy a_t at each date t, fully anticipating that his authority is perpetual. We refer to this as *permanent authority (PA)*

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[6] This implies that the distribution of population characteristics is summarized by a finite dimensional parameter set.

[7] In fact, an earlier version of the paper introduced the model with additive stochastic shocks of the form $\omega_{t+1} = \nu_{t+1} + Q(\omega_t, a_t)$ with $\{\nu_t\}$ iid across states and across time. We took them out to simplify notation since they added nothing to the qualitative features of the model.

regime. PA regimes are not, almost by definition, common in modern democracies. They were common, however, in many European monarchies prior to the 19th century.

Significantly, most political economy models either assume explicitly a PA regime (e.g., a social planner) or derive one in equilibrium under special assumptions on preferences, technology, and political institutions. In either case, the PA regime is the natural benchmark against which to compare a model of policy-endogenous power produced by Faustian trade offs.

Under permanent authority, the ruler i_0 choices may be characterized by a policy function ψ in which $\psi(\omega_t) = a_t$ is the policy that would be taken by citizen-type i_0 in state ω_t The policy function ψ solves i_0's Bellman equation

$$V(i_0, \omega_t; \psi) \; = \; \max_{a_t \in A} \; [u(i_0, \omega_t, a_t) \; + \; \delta \; V(i_0, \omega_{t+1}; \psi)] \tag{2}$$

subject to $\omega_{t+1} = Q(\omega_t, a_t)$. This problem is standard and serves as the benchmark case. We will refer to any solution (2) as a *Permanent Authority (PA)* equilibrium.

2.3 Policy-Endogenous Political Power

We contrast the PA case with an environment with *policy endogenous (PE) political power.* Policy-endogenous power is a regime in which policy choices induce changes in political power. To focus on decision-theoretic aspects, political power is modeled in reduced form by assuming that the political system is rationalized by the preferences of a pivotal individual. Specifically, in each period, the political institution produces a policy that coincides with the preferred choice of a particular citizen. This citizen (e.g., pivotal voter or political leader) is, in effect, endowed with the exclusive right in period t to choose the policy action a_t. Henceforth, we refer to this individual as the *leader.* Political power is therefore represented by a mapping from states (e.g., capital stocks, income distributions) to citizen-types. Formally, the mapping is assumed to be a smooth, weakly monotone (increasing or decreasing) function $\mu : \Omega \rightarrow I$. Given a state ω_t, the leader is a citizen-type $i_t = \mu(\omega_t)$.

Because μ determines "who's in charge" in each state, we refer to it as the *authority function.* The assumption that the political process admits a pivotal leader clearly involves some loss of generality. Voting aggregation, for instance, is not so well behaved when policies are multi-dimensional.[8]

The key attribute of an authority function, for our purposes, is that it admits the possibility that political power changes endogenously due to changes in the state. Current policy changes produce changes in the state which, in turn, produce changes in the identity of the

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[8] There are, however, commonly assumed conditions on policy preferences, notably single crossing properties (see Gans and Smart (1996) and Rothstein (1990)) that do admit pivotal voters.

leader through μ. Let $i_t = \mu(\omega_t)$ denote the leader in state ω_t. The change in authority, as described by μ, defines a dynamic stochastic game with a potentially infinite set of players. The players' choices result in a stationary Markov process that realizes states $\{\omega_0, \omega_1, \omega_2, \ldots, \}$, leaders $\{i_0, i_1, i_2, \ldots, \}$, and policies $\{a_0, a_1, a_2, \ldots\}$. We refer to these processes as the *Faustian dynamics* of PE political power.

The underpinnings of PE power is open to interpretation. We have in mind a polity summarized by a fixed set of (*de jure*) rules, with *de facto* political power evolving naturally in economic state. There are two natural interpretations of how this might happen.

Demographic channels. The state ω_t directly identifies the population distribution or the voter turnout of citizen types. Changes in, say, immigration laws or fertility policy can have (longer run) electoral effects even under median voter rules, since the demographic changes themselves may be biased toward certain groups or social classes. For example, Tichenor (2002) describes 19th century immigration policy in the U.S., sometimes restrictive but more often expansive, that ultimately brought about large political shifts toward urban regions that came to be reflected in congressional and presidential elections.

Distributional channels. The state ω_t identifies temporal characteristics such as wealth, income, or social status of the citizen-types, and the polity depends on cardinal changes in these characteristics. Tax cuts, for instance, may change the identity of the pivotal voter in a heterogeneous economy if campaign contributions affect the outcome of an election.

In both channels, the Faustian trade-off results from a "bias" in the political system. In some cases, political bias is explicitly built in to the polity as in the case of the U.S. Senate. In other cases, the bias is implicit such as when a citizen's political influence depends on his wealth.[9]

To illustrate how this can happen, suppose that a type i citizen has $y(i, \omega_t)$ in state ω_t, and that higher i-types have lower income in each state. Assuming a uniformly distributed population on $[0, 1]$ and a standard "Intermediate preference" assumption on period payoffs (Grandmont (1977)), it is not hard to show that majority voting implies $\mu(\omega_t) = 1/2$ regardless of the state.

Suppose instead that the polity allocates $y(i, \omega_t)$ votes to each type i. In other words, voting is weighted by one's wealth.[10] Then (again assuming intermediate preferences), μ is

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[9] Even cases where wealth matters, the bias is not automatic. To illustrate, consider a polity in which influence is auctioned off to the highest bidder. Since policy changes typically preserve first order stochastic dominance in relative income, the identity of the highest bidder (in a monotonic equilibrium) is constant through time. What matters is that the polity vary with cardinal rather than ordinal properties of the income distribution.

[10] Such a polity is not exotic. Wealth-weighted systems were common in Europe in the 19th century. Germany, for instance, allocated vote shares using the percentage of taxes paid. The electorate was divided into thirds, each third given equal weight in the voting. According to Finer (1997), the wealthiest 3.5% of the

endogenously determined by

$$\frac{\int_{\mu(\omega_t)}^{1} y(i,\omega_t)di}{\int_0^1 y(i,\omega_t)di} = \frac{1}{2}$$

Hence, while order-preserving shifts in the income distribution do not change the median voter, they do change the wealth-weighted pivotal voter. In this case, μ increases (decreases) whenever Lorenz inequality increases (decreases) in the state ω_t.

For the remaining analysis, we take the authority function μ as a given, and explore the dynamic consequences of policy-endogenous power. However, as the above example suggests, authority functions can be derived from explicit voting procedures. A recent model of Campante (2007) also shows how it arise from campaign contributions.

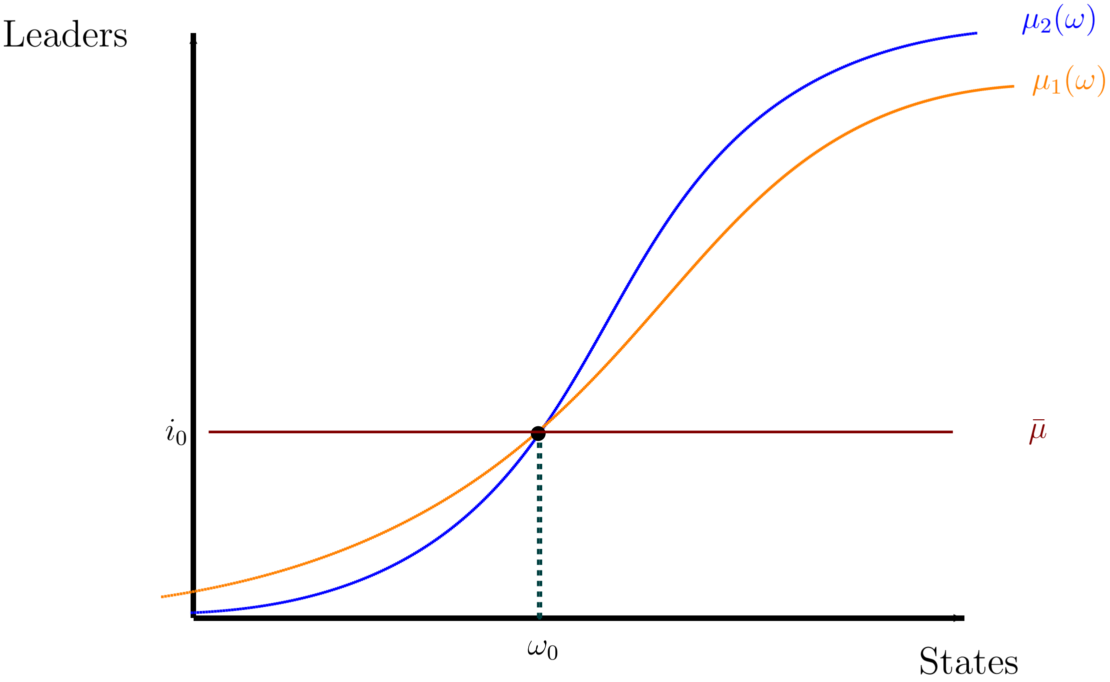

Figure 1: Authority Functions in the Policy-Endogenous and Permanent Power Regimes

Figure 1 illustrates various political systems modeled as authority functions, each with differing degrees of "bias." The fully unbiased system is $\bar{\mu}$, corresponding to a permanent authority (PA) regime with i_0 as the permanent leader. Clearly there is no Faustian trade-off for i_0 in this case. In the sequel, we fix i_0 as both the permanent authority and the *initial* decision maker under policy-endogenous power. Consequently, the difference in between a PE

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population was allocated one third of the votes, another 10-12% of the population was allocated the second third, and the remainder accounted for the final third.

and PA path, or between two PE paths, comes from different political institutions rather than different initial states.

In the Figure, μ_1 and μ_2 both begin with i_0 as the initial decision maker. The function μ_1 is "less biased" than μ_2 in the sense that μ_2 offers a starker Faustian trade-off for leader i_0. Intuitively, one might call μ_1 more "conservative" in the sense of offering a more gradual evolution of political power; μ_2 is more "progressive" in the sense inducing accelerated change.

2.4 Equilibrium

Under PE power generally, the leader i_t in period t chooses an action a_t anticipating the effect that it has on the future trajectory of states, leaders, and policies. In principle, this choice could be contingent on the entire history of the game. However, we restrict attention to Markov Perfect equilibria (i.e., Subgame Perfect equilibria in Markov strategies). There are two reasons for this. First, Markov strategies facilitate a direct comparison with the permanent authority equilibrium in (2) which is inherently Markov. Second, we think the restriction is a natural one. With a large population of decision makers, the costs of coordination are presumably higher. Strategies that therefore depend only on the current, payoff relevant state bypass some of the difficulty.

Let Ψ^* denote a Markov strategy mapping states to policies. Formally, $a_t = \Psi^*(\omega_t)$ is the policy chosen in state ω_t. The continuation payoff to a citizen-type i in date t under a Markov strategy Ψ^* is

$$V(i, \omega_t; \Psi^*) \;=\; \sum_{\tau=t}^{\infty} \delta^{\tau-t}\; u(i, \omega_\tau, \Psi^*(\omega_\tau)) \tag{3}$$

Given the sequence $\{\omega_t\}$ induced by transition function Q.

We define a *Policy-Endogenous (PE) equilibria* as a Markov Perfect equilibrium in the PE environment. More precisely, a PE equilibrium is a Markov profile Ψ^* such that Ψ^* is a best response by citizen-type i against any history-contingent strategy that differs from Ψ^* only in states ω for which $\mu(\omega) = i$. A standard argument shows that the so-called *one-shot deviation principle* applies. Namely, that Ψ^* is a PE equilibrium if and only if for all ω_t, and for all a_t,

$$V(\mu(\omega_t), \omega_t; \Psi^*) \;\geq\; u(\mu(\omega_t), \omega_t, a_t) \;+\; \delta\; V(\mu(\omega_t), Q(\omega_t, a_t); \Psi^*) \tag{4}$$

To expand further, consider what policy *would have been chosen* in state ω_t by an arbitrary citizen-type i in a PE equilibrium? This question is hypothetical because an arbitrary i is not the leader in state ω_t unless it happens that $i = i_t = \mu(\omega_t)$. Let $a_t = \Psi(i, \omega_t)$ denote this hypothetical decision. Ψ is a *hypothetical Markov strategy* mapping types and states into policies. The hypothetical decision is optimal if for all a_t

$$u(i, \omega_t, \Psi(i, \omega_t)) \;+\; \delta\; V(i,\; Q(\omega_t, \Psi(i, \omega_t)); \Psi^*) \;\geq\; u(i, \omega_t, a_t) \;+\; \delta\; V(i, Q(\omega_t, a_t); \Psi^*) \tag{5}$$

The inequality in (5) coincides with (4) in those states for which $i = \mu(\omega_t)$. The hypothetical strategy and the PE equilibrium are therefore related by $\Psi(\mu(\omega_t), \omega_t) \ = \ \Psi^*(\omega_t)$.

Hence, starting from initial state ω_0, the decision maker i_0 chooses $\Psi(i_0, \omega_0) \equiv \Psi^*(\omega_0)$. Type i_0 correctly anticipates that his chosen policy a_0 leads to a possible change in decision authority at date $t = 1$. This change is given by $i_1 = \mu(\omega_1)$ where next period's state ω_1 is determined by $\omega_1 = Q(\omega_0, \Psi^*(\omega_0))$. It is generally true that $\Psi(i_0, \omega_1) \neq \Psi^*(\omega_1)$ because i_0 no longer makes the decision in state ω_1 in the PE equilibrium, and the decision type i_1 will generally have different preferences over policy. The more interesting comparison, however, is between $\Psi(i_0, \omega_1)$ and $\psi(\omega_1)$. It tells us how an arbitrary citizen-type would react to the loss of power in a given state, where the extent of the loss is determined by that state.

In certain instances, the current leader faces no Faustian trade-off even when μ exhibits bias. For instance, suppose that period payoffs are of the form $u(i, \omega, a) = u_1(i) + u_2(\omega, a)$ or the form $u_1(i)u_2(\omega, a)$. In either case, the additive or multiplicative separability implies that individual-specific characteristics do not affect policy preferences. Individuals' policy preferences are therefore the same, and so changes in power have no effect on policy.

3 A Parametric Model of Public Investment

This Section examines a canonical special case based on a consumption-leisure model similar to a setup used by Battaglini and Coate (2007). The purpose here is to illustrate how the abstract model can be generated by familiar primitives (in this case, primitives familiar to macroeconomists), and how Faustian dynamics work in such a model. In addition, the example provides some implications of independent interest on the interactive effects of growth, inequality, and the evolution of power. We later show that most of the qualitative features of this equilibrium arise generally in wide class of smooth, monotone PE models.

In each period, each citizen is endowed with one unit of time which he divides between private and public sector effort. Private sector effort ℓ_{it} generates income $g(i)\ell_{it}$. Public sector effort a_t is viewed as uniform or lump sum contribution $a_t \in [0, \bar{a}]$ of time (such as compulsory military service) that society collects from each citizen each period. The sum of these public contributions produces a public capital good, ω_t, such as defense, public education, infrastructure, which is accessible to all citizens, each of whom can use it to augment his own wealth over time. Assume

$$\omega_{t+1} \ = \ (1 - d)\omega_t \ + \ a_t, \tag{6}$$

where $d \in (0, 1]$ is the depreciation rate on the current stock of public capital. Citizen then i consumes c_{it} from his income $y\left(i, \ell_{it}, \omega_t\right) \ = \ g\left(i\right)\ell + \omega$. There is no private saving.[11] Finally,

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[11] Hence, income and wealth are equivalent concepts. Consequently we use "income" and "wealth" interchangeably.

the citizen's utility is given by $\tilde{u}\left(i, c_{it}, a_t, \ell_{it}\right) \ = \ c_{it} \ - \ \frac{\ell_{it}^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \ - \ \frac{1}{f(i)}\frac{a_t^2}{2}$ where $f' > 0$.

Each citizen therefore allocates his time optimally between private and public sector labor, solving

$$\max_{0 \leq \ell \leq 1} \left(g\left(i\right)\ell + \omega - \frac{\ell^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \right) - \frac{1}{f\left(i\right)}\frac{a^2}{2}. \tag{7}$$

Notice that the private maximization part only involves the first term in the bracket, which is exactly the same as Battaglini and Coate (2007). Substituting the solution $\ell^*\left(i\right) = \left(g\left(i\right)\right)^{\varepsilon}$ into the objective in (7) produces an indirect indirect utility function of the form $u(i, \omega_t, a_t)$. By dropping the constant term, we derive our previous u as the indirect utility

$$u\left(i, \omega_t, a_t\right) \ = \ f\left(i\right)\omega_t - \frac{a_t^2}{2}. \tag{8}$$

Two brief remarks: first, idiosyncratic productivity $g(i)$ drops out of (8) as it plays no role in citizen i's policy decision. However, $g(i)$ may play an indirect role in the evolution of the economy through it's effect on μ. In this sense, income inequality generated by differences in g can matter. Second, because $f' > 0$ higher types have higher marginal value government capital. Therefore, higher i-types may be regarded as more " fiscally liberal."

The political institution is summarized by an authority function given by

$$i_t \ = \ \mu(\omega_t) \ \equiv \ f^{-1}(\kappa_0 + \kappa\omega_t), \quad \kappa_0 > 0 \tag{9}$$

Here, the authority is given to the type i_t for whom the marginal value for the public capital is an affine function of the capital stock itself. By varying parameters κ and κ_0 such that all authority functions intersect the initial point (ω_0, i_0) (see Figure 1), Equation (9) maps out a one-dimensional class of authority functions, each of which differs by the adjustment speed and direction of political power.[12] Other things equal, larger $|\kappa|$ (adjusting κ_0 to keep the same initial point) implies faster evolution of political power, i.e., less gradual adjustment of political authority over time.

When $\kappa > 0$, μ is increasing. An increase in public capital therefore adjusts political power upward toward more fiscally liberal types — those with higher marginal value of government expenditure. But more liberal types choose higher levels of government expenditure which increase the public capital stock. In this sense, μ represents a *reinforcing political bias.* When $\kappa < 0$, an increase in public capital adjusts political power downward, toward the more conservative "small government" types. Because the authority function μ moves in opposition to the transition technology, it represents a *countervailing political bias.* The special case of $\kappa = 0$ corresponds to the familiar case of permanent authority, hence no bias.[13]

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[12] More precisely, this class of authority functions are those that satisfy $\mu(\omega_t) \ = \ f^{-1}(\kappa_0 + \kappa\omega_t)$ such that κ and κ_0 satisfy the linear equation $f(i_0) = \kappa_0 + \kappa\omega_0$.

[13] Because $f\left(i\right)$ is an increasing function, the Permanent Authority equilibrium can be identified as a solution

3.1 Monotone Faustian Dynamics

Consider first the case of $\kappa \geq 0$, thus a reinforcing bias. Since μ slopes upward, increased public investment places power in the hands of more fiscally liberal citizen-types.

Proposition 1 *Consider any authority function μ satisfying (9). If $0 \leq \kappa < d\left(\frac{1}{\delta} - 1 + d\right)$, then the following hold.*

(i) There exist a unique, strictly increasing PE equilibrium policy rule $\Psi^*(\omega_t)$, a unique, strictly decreasing hypothetical rule $\Psi(i_0, \omega_t)$, and a unique stationary PA equilibrium $\psi(\omega_t) = \bar{\psi}$.

(ii) The PE equilibrium path of states $\{\omega_t\}$ converges monotonically to a unique steady state $\bar{\omega}$, and if $\omega_0 < \bar{\omega}$, then for all $\omega_t > \omega_0$,

$$\Psi(i_0, \omega_t) < \psi(\omega_t) \;\; \text{and} \;\; \Psi(i_0, \omega_t) < \Psi^*(\omega_t).$$

(iii) There exists a state ω^* with $\omega_0 < \omega^* < \bar{\omega}$ such that

$$\begin{array}{rcll} \Psi^*(\omega_t) & < & \psi(\omega_t) & \forall \omega_t < \omega^* \\ \Psi^*(\omega_t) & \geq & \psi(\omega_t) & \forall \omega_t \geq \omega^* \end{array}$$

The proof of the Proposition in the Appendix first "guesses" a particular affine policy function of the form $\Psi^*(\omega_t) = A^* + B^*\omega_t$ then verifies its form and finds the coefficients $A^* > 0$ and $0 \leq B^* < d$ as implicit solutions to equations that satisfy the implied value function.[14] Properties of the implicit solutions for the coefficients may then be used to prove the "comparative statics" results in (ii) and (iii).

Parts (ii) and (iii) state what turn to be quite general properties of Faustian dynamics. Part (ii) compares the hypothetical PE rule to both the PE and PA rules. It asserts that type i_0 is always more conservative in his decision making under policy-endogenous power than he would be if his power were permanent, and is more conservative than the realized leader in the PE equilibrium. In fact, the (hypothetical) policies of leader i_0 become more conservative over time/states because as the state progresses upward, more distant types assume power.

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to a social planner's problem. To see this, suppose that h is a density on $[0,1]$ in which $h(i)$ is the welfare weight assigned to citizen-type i. Then there exists i_0 such that

$$f(i_0) = \int h(i) f(i)\, di.$$

In other words, the social welfare function coincides with the utility function of a specially chosen i_0. Therefore, the social planner's problem is the same as the PA problem with permanent authority vested in type i_0.

[14] The coefficients A^*, B^* implicitly vary in the exogenous parameters κ_0, κ, δ, and d.

But, as Part (iii) shows, individual caution is juxtaposed with progressive evolution of policy. The PE equilibrium starts out more conservative than PA, but is more progressive in long run. The intuition is as follows. The decision maker i_0 anticipates a growing economy under Ψ^*; he also anticipates the corresponding shift of power to higher marginal value type in the future. Because a higher type is more liberal, i.e., has a higher marginal value of investment, the future contribution will be higher than the desired level of the current decision maker. By choosing a lower current contribution now, i_0 decreases the gap between the chosen policies of future leaders and the policies that he would have preferred. Type i_0 therefore reduces his current investment, thereby slowing the transition of power toward more progressive types. The fact that $\Psi(i_0, \omega_t)$ is decreasing in the state demonstrates, in fact, that the incentive to "slow things up" intensifies over time. See Figure 2.

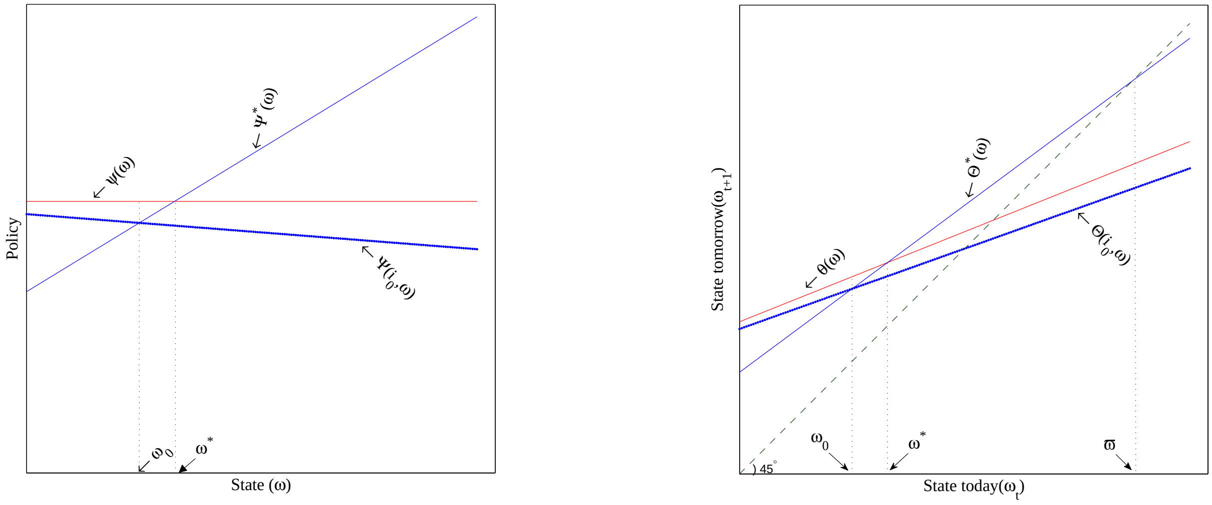

Figure 2: Monotone Dynamics in the PE and PA Equilibria.

The motive for "political preservation" can be precisely identified in a leader's Euler equation. Using the parametric assumptions, the value function for an arbitrary type i is

$$V\left(i, \omega_t;\ \Psi^*\right) \ = \ \max_{a \in A} \ \left\{ f(i)\omega_t - \frac{a^2}{2} \ + \ \delta\ V\left(i, \omega_{t+1};\ \Psi^*\right) \right\} \quad \text{subject to} \quad \omega_{t+1} = (1-d)\,\omega_t + a \tag{10}$$

If $\Psi^*(\omega_t)$ lies in the interior of A, then it satisfies the first-order condition[15]

$$0 \ = \ -\Psi^*(\omega_t) + \delta D_{\omega_{t+1}} V\left(i_t, (1-d)\,\omega_t + \Psi^*(\omega_t);\ \Psi^*\right). \tag{11}$$

The first term is clearly the marginal cost of an increase in government spending, while the second is the discounted marginal benefit in the future. Consider next period's decision from

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[15] Throughout the paper, the partial derivative of a function $G(x, y)$ is expressed as $D_x G$.

the point of view of the *current* decision maker i_t. Since next period's decision maker i_{t+1} is different from i_t, the decision next period induces a marginal distortion away from i_t's optimal policy choice in $t+1$. This distortion is given by

$$\Delta(i_t, \omega_{t+1}; \Psi^*) \;=\; -\Psi^*(\omega_{t+1}) + \delta D_{\omega_{t+2}} V\left(i_t, (1-d)\,\omega_{t+1} + \Psi^*(\omega_{t+1});\; \Psi^*\right). \tag{12}$$

Notice that the right-hand side of (12) is of the same form as (11), shifted one period ahead. Intuitively, because the government's capital stock ω_t increases, power shifts toward more progressive policy makers who prefer toward higher levels of government spending. Hence, $\Delta(i_t, \omega_{t+1}; \Psi^*) < 0$ since i_{t+1} chooses a higher level of spending than i_t himself would choose in state ω_{t+1}.

Differentiating the value function V in (10), substituting in the distortion equation (12), and iterating one period, the discounted marginal benefit $D_{\omega_{t+1}} V\left(i, \omega_{t+1};\; \Psi^*\right)$ of an increase in current investment now be expressed as

$$\begin{aligned} D_{\omega_{t+1}} V\left(i, \omega_{t+1};\; \Psi^*\right) \;&=\; \left[f\left(i\right) + \left(1-d\right)\left(A^* + B^*\omega_{t+1}\right)\right] \quad + \quad \left[\left(B^* + 1 - d\right)\Delta(i, \omega_{t+1};\; \Psi^*)\right] \\ &\equiv\; R\left(i, \omega_{t+1}; \Psi^*\right) \quad + \quad P\left(i, \omega_{t+1}; \Psi^*\right) \end{aligned} \tag{13}$$

The marginal continuation value $D_{\omega_{t+1}} V$ can be decomposed into two effects. The first, $R\left(i, \omega_{t+1}; \Psi^*\right)$, is standard in all dynamic decision problems. It describes the direct effect that current policy has on next period's state. The second effect, $P\left(i, \omega_{t+1}; \Psi^*\right)$, is dubbed the *political preservation effect* since it captures the marginal cost imposed on the leader by the endogenous change in future leaders. This cost induces more conservative policies in the sense of slowing the evolution of political power as it moves away from the current leader.

Because the leader i_0's PE policy choice may be well below his PA policy, it takes time before the PE path overtakes that of permanent authority. Part (iii) shows that this eventually happens. This is illustrated in Figure 2. Hence, for small ω_t, the leader-type is not so different from i_0, and so the individual's preference for conservative change largely determines the ordering between Ψ^* and ψ. As the leader-type evolves, eventually moving away from i_0, the evolution preference types determines the comparison.

The Faustian dynamics of states and leaders also moves progressively. To see this, observe that since ω_0 is close to zero, it lies below the steady state. Hence, the equilibrium state transition $\Theta^*(\omega_t) \equiv Q(\omega_t, \Psi^*(\omega_t))$ is increasing in the state, and, consequently, the equilibrium paths $\{\omega_t\}$ and $\{i_t\}$ are increasing. This is illustrated in the second graph in Figure 2 which displays the comparison between the PE transition Θ^*, the hypothetical transition Θ, and the PA transition θ.

3.2 Cyclical Faustian Dynamics

Now consider $\kappa \leq 0$, the case of countervailing bias. Since μ slopes downward, increased public investment places power in more politically conservative citizen-types.

Proposition 2 *Consider any authority function* μ *satisfying (9). If* $0 \geq \kappa \geq -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta}\left[(1-d)^2 + \frac{3+\delta}{1+\delta}(1-d) + \frac{1}{\delta}\right]$*, then the following hold.*

(i) There exists a unique, decreasing PE equilibrium policy rule $\Psi^*(\omega_t)$.

(ii) The PE equilibrium path of states $\{\omega_t\}$ *converges to a unique steady state* $\bar{\omega}$. *If* $\kappa > -\frac{1-d}{\delta}$*, then the convergence is monotonic:* $\omega_{t+1} > \omega_t$. *However, if* $\kappa < -\frac{1-d}{\delta}$*, then the economy follows a dampened cycle converging to* $\bar{\omega}$ *such that* $\omega_t < \bar{\omega}$ *if and only if* $\omega_{t+1} > \bar{\omega}$. *In either case,*

$$\Psi^*(\omega_0) = \Psi(i_0, \omega_0) < \psi(\omega_0) \quad \forall \omega_0 \leq \bar{\omega}, \quad \textit{and}$$

$$\Psi^*(\omega_0) = \Psi(i_0, \omega_0) > \psi(\omega_0) \quad \forall \omega_0 > \bar{\omega}$$

Faustian dynamics can therefore produce political cycles when the system is sufficiently biased downward. The intuition, roughly, is that the evolution of political power counters the evolution of government capital. Hence, when fiscally liberal types choose high expenditures, leading to increases in capital stock, this induces a steep drop in the index i that determines the progressivity of the political type.[16] More fiscally conservative types then lower expenditures which, in turn, produce more liberal types, and so on.

Since μ slopes downward, then whenever the government's capital stock increases, political power moves downward toward more fiscally conservative types. Consequently, the preservation effect induces the initial, liberal, leader to decrease expenditures and hence *slow* the evolution of political authority as it moves downward. See Figures 3 and 4.

3.3 Institutional Comparisons

So far, we have compared the PE to the PA regime. This comparison, however, is just a special case of comparing two PE political institutions, one yielding more gradual change in political power than the other. This was illustrated in Figure 1.

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[16] With a countervailing bias, there is a difference between "liberal" ("conservative") in the traditional sense and "fiscally liberal" ("fiscally conservative"). The former refers to a desire to speed up (slow down) the process of political change, while the latter refers to the preference for larger (smaller) government expenditures.

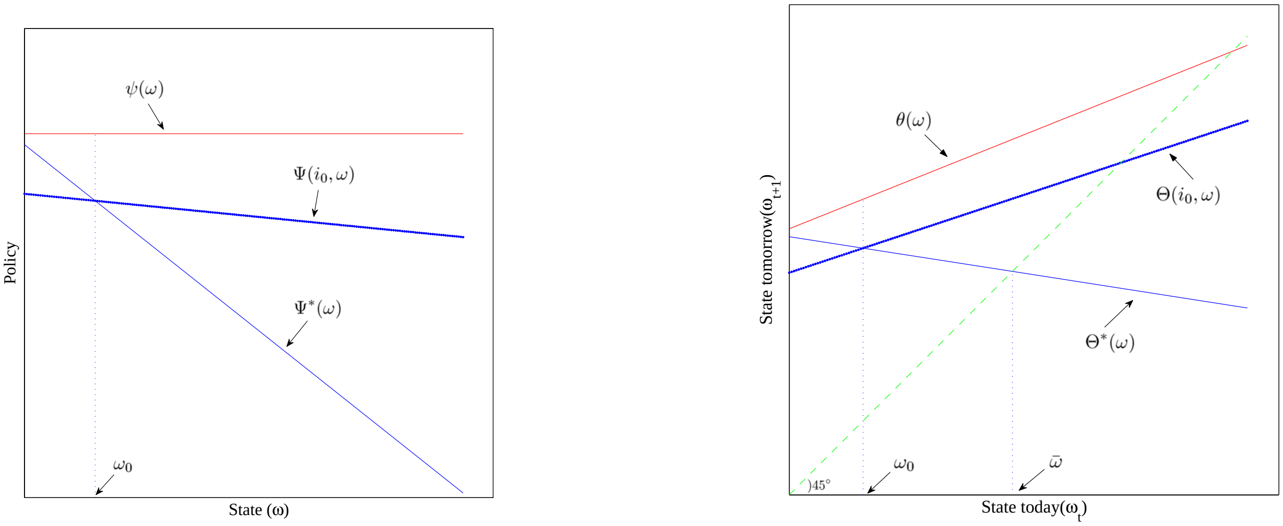

Figure 3: Cyclical Dynamics in the PE and PA Equilibria.

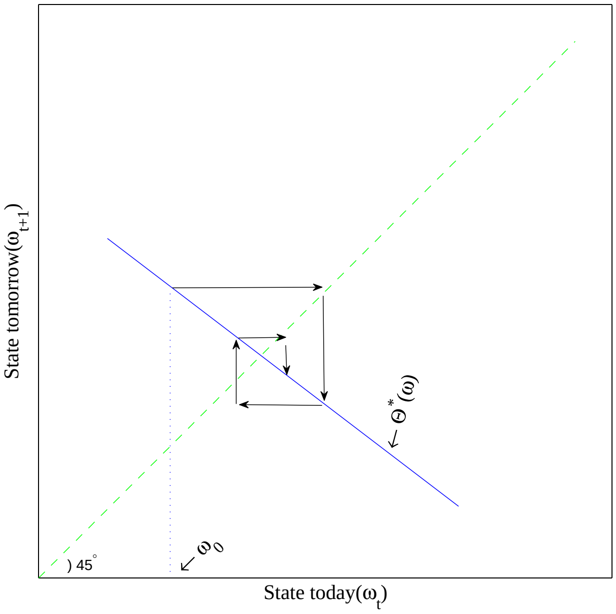

Figure 4: Cycles with Countervailing Bias

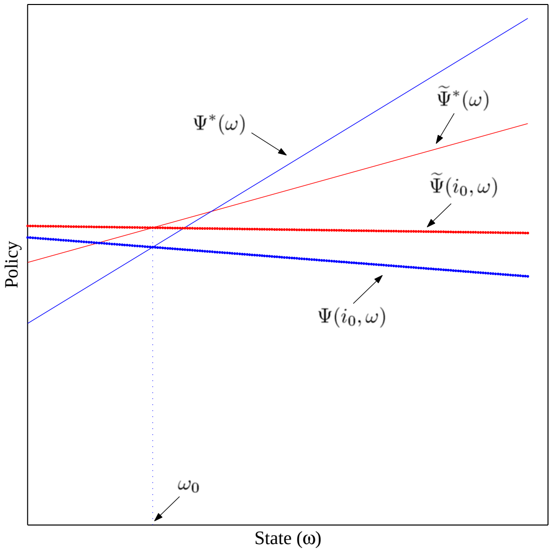

Figure 5: Comparison of Two Political Institutions

Proposition 3 *Consider two authority functions* μ *and* $\tilde{\mu}$*, each corresponding to a parameter pair* (κ_0, κ) *and* $(\tilde{\kappa}_0, \tilde{\kappa})$ *according to (9), and both of which satisfy the initial condition. Suppose* $\kappa > \tilde{\kappa}$ *(and, consequently,* $\kappa_0 < \tilde{\kappa}_0$*) and* $\kappa\tilde{\kappa} > 0$*. Let* Ψ^* *and* $\widetilde{\Psi}^*$ *denote PE equilibria under* μ *and* $\widetilde{\mu}$ *respectively, and let* $\bar{\omega}$ *denote the unique steady state under* Ψ^**. If* $\omega_0 < \bar{\omega}$ *then there exists a state* ω^* *satisfying* $\omega_0 < \omega^* < \bar{\omega}$ *and*

$$\begin{array}{rcll} \Psi^*(\omega_t) & < & \widetilde{\Psi}^*(\omega_t) & \forall \omega_t < \omega^* \\ \Psi^*(\omega_t) & \geq & \widetilde{\Psi}^*(\omega_t) & \forall \omega_t \geq \omega^* \end{array}$$

Recall that if $\kappa > \tilde{\kappa}$, then μ gives a faster evolution of political power, while $\tilde{\mu}$ is more gradual. The intuition is the same as in Part (iii) of Proposition 1. Decision makers respond to a more biased political institution μ, by choosing *more* conservative responses. Hence, the more biased institution μ initially produces a slower evolution of power and policy than the less biased one $\tilde{\mu}$. However, in the long run, structural features take over, and the more biased polity produces a faster evolution of leaders and policies. This is illustrated in Figure 5.

4 A General Monotone Model

This Section returns to the general framework. For tractability, we restrict our focus to increasing authority functions. Under supermodular assumptions given below, this amounts to assuming that the bias is reinforcing. As before, our goal is to compare the PE equilibrium Ψ^*, the PA equilibrium ψ, and the hypothetical rule $\Psi(i_0, \cdot)$.

The main results characterize properties of smooth-limit equilibria, which we define below. A natural decomposition of the Euler equation is established in which the "political preservation effect" and the "reformation effect" which were defined for the parametric model, can be identified in the general model. These effects are used to compare the equilibrium trajectory of policy and power with that of the permanent authority. The proofs of all results are in the Appendix.

Smoothness (differentiability) plays a crucial role in our characterization. We use it to examine properties of the Euler equations, roughly following an approach dating back to Basar and Olsder (1982) for dynamic stochastic games.[17] Issues of equilibrium existence are not explored here. There are, in fact, general existence results for smooth Markov equilibria, but these require stochastic shocks which are absent from our presentation.[18]

4.1 Distortion Adjusted Euler Equation

A PE equilibrium Ψ^* and PA equilibrium ψ are *smooth-limit equilibria* if (i) Ψ^* and ψ are differentiable in the state; and (ii) the resulting policies and $\Psi^*(\omega_t)$ and $\psi(\omega_t)$ lie in the interior of the feasible policy space, A, and (iii) Ψ^* and ψ are the limit of smooth, finite horizon PE and PA equilibria, resp. Property (iii) is not necessary in the following characterization of the Euler equation. It is only used later when time-iteration of the value function is the most convenient way of establishing monotonicity in the state.

The PE and PA first-order conditions (resp.) are:

$$D_{a_t} u(\mu(\omega_t), \omega_t, \Psi^*(\omega_t)) + \delta\, D_{a_t} Q(\omega_t, \Psi^*(\omega_t)) \cdot D_{\omega_{t+1}} V(\mu(\omega_t), Q(\omega_t, \Psi^*(\omega_t));\ \Psi^*) \;=\; 0 \tag{14}$$

and

$$D_{a_t} u(i_0, \omega_t, \psi(\omega_t)) \;+\; \delta\, D_{a_t} Q(\omega_t, \psi(\omega_t)) \cdot D_{\omega_{t+1}} V(i_0, Q(\omega_t, \psi(\omega_t));\ \psi) \;=\; 0 \tag{15}$$

In these equations, $D_{a_t} u$ is a $\ell \times 1$ gradient vector of partial derivatives in a_t, $D_{\omega_{t+1}} V$ is a $\ell \times 1$ gradient vector of partial derivatives in ω_{t+1}, and $D_{a_t} Q$ is a $\ell \times \ell$ gradient matrix of partial derivatives in a_t.

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[17] More recently this approach has been adapted to dynamic macro policy problems by Klein, Krusell, and Rios Rull (2002), Krusell and Smith (2003), Krusell, Kuruscu, and Smith (2002), and Judd (2004), and to dynamic political games by Jack and Lagunoff (2004).

[18] A previous draft of this paper had shocks, but we took them out as they added little to the main ideas. Shocks aside, our assumptions do not appear to violate any of the known existence results as far we are aware. See, for instance, Amir (1996), Curtat (1996), Horst (2005), Lagunoff (2006b), and Novak (2007).

Consider an arbitrary citizen-type $i \in I$ (not necessarily the leader) in the PE equilibrium. His continuation value function in state ω_{t+1} is

$$V(i, \omega_{t+1}; \Psi^*) \;=\; u(i, \omega_{t+1}, \Psi^*(\omega_{t+1})) \;+\; \delta\, V(i, \omega_{t+2}; \Psi^*) \tag{16}$$

with $\omega_{t+2} = Q(\omega_{t+1}, \Psi^*(\omega_{t+1}))$. To save on notation, we use the abbreviated notation $u_i = u(i, \cdot)$, and $V_i = V(i, \cdot)$ to denote the stage and dynamic payoffs, respectively, of a type i. Differentiating this value function V_i with respect to ω_{t+1} yields

$$D_{\omega_{t+1}} V_i \;=\; D_{\omega_{t+1}} u_i \;+\; D_{\omega_{t+1}} \Psi^* \cdot D_{a_{t+1}} u_i \;+\; \delta \left[D_{\omega_{t+1}} Q \;+\; D_{\omega_{t+1}} \Psi^* \;\cdot\; D_{a_{t+1}} Q \right] \cdot D_{\omega_{t+2}} V_i \tag{17}$$

When this citizen *is* the leader, i.e., if $i = i_t = \mu(\omega_t)$ in state ω_t, and if $D_{a_t} Q$ is invertible then i_t's first order condition in (14) can be rewritten as

$$\delta D_{\omega_{t+1}} V_{i_t} \;=\; -\left[D_{a_t} Q\right]^{-1} \cdot D_{a_t} u_{i_t} \tag{18}$$

Now recall, from the previous Section, the *distortion function* of a citizen-type i:

$$\Delta(i, \omega_t; \Psi^*) \;=\; D_{a_t} u(i, \omega_t, \Psi^*(\omega_t)) \;+\; \delta\, D_{a_t} Q(\omega_t, \Psi^*(\omega_t)) \cdot D_{\omega_{t+1}} V(i, \omega_{t+1};\; \Psi^*) \tag{19}$$

with $\omega_{t+1} = Q(\omega_t, \Psi^*(\omega_t))$. In the general model, the distortion function $\Delta(i, \omega_t; \Psi^*)$ is a $\ell \times 1$ gradient vector that describes the marginal payoff deviation from i's own preferred policy, due to the fact that i is possibly not the leader in the state ω_t. When i *is* the leader, i.e., when $i = i_t = \mu(\omega_t)$, then $\Delta(i_t, \omega_t; \Psi^*) = 0$, i.e., the distortion is zero. Of course, by definition, it also follows that $\Delta(i_0, \omega_t; \psi) = 0$ since there is no distortion when i_0 holds power forever.

The distortion equation (19) can be written as

$$\delta D_{\omega_{t+1}} V_i \;=\; \left[D_{a_t} Q\right]^{-1} \cdot \left[\Delta(i, \omega_t; \Psi^*) \;-\; D_{a_t} u_i\right] \tag{20}$$

Iterating (20) forward one period while holding indexed i fixed, and then substituting it back into (17) yields

$$\begin{aligned} D_{\omega_{t+1}} V_i \;&=\; D_{\omega_{t+1}} u_i \;+\; D_{\omega_{t+1}} Q \cdot \left[D_{a_{t+1}} Q\right]^{-1} \cdot \left[-D_{a_{t+1}} u_i\right] \\ &\quad +\; \left[D_{\omega_{t+1}} Q \cdot \left[D_{a_{t+1}} Q\right]^{-1} \cdot \left[\Delta(i, \omega_{t+1}; \Psi^*)\right] \right] \;+\; D_{\omega_{t+1}} \Psi^* \;\cdot\; \Delta(i, \omega_{t+1}; \Psi^*) \end{aligned} \tag{21}$$

At this point, the expression (21) cannot be simplified without further structure. We employ the following assumptions throughout the remainder of the analysis.

(A1). (Invertible Transition) Q is invertible in the policy variable, and its inverse $L(\omega, \omega') \equiv Q_{\omega}^{-1}(\omega') \;=\; a$ is increasing in ω' and decreasing in ω.

(A2). (Supermodularity) Q is supermodular in vector (a, ω).[19] u is supermodular in (i, ω, a), and the indirect payoff function $u(i, \omega, L(\omega, \omega'))$ is supermodular in (i, ω).

Assumption (A1) is used to express (21) in a more useful form. While it restricts the general model, it is also satisfied by a number of natural models such as the separable transition $Q(\omega, a) = (Q_1(\omega, a_1), \ldots, Q_\ell(\omega, a_\ell))$. Here, each capital stock ω_k is affected only by a distinct policy a_k.

Assumption (A2) is a collection of monotone comparative statics conditions.[20] As with all such assumptions, Assumption (A2) allows us to make monotone comparisons between policy-endogenous and permanent authority regimes. The last part of the assumption requires that u is supermodular (i, ω) even after accounting for the effect on u through cost function L.[21]

For now, only Assumption (A1) is needed. First, define the functions $\Theta(i, \omega)$ and $\Theta^*(\omega)$ by

$$\Theta(i, \omega) \;=\; Q(\omega, \Psi(i, \omega)) \quad \text{and} \quad \Theta^*(\omega) \;=\; Q(\omega, \Psi^*(\omega)) \tag{22}$$

The function $\Theta^*(\omega_t) = \omega_{t+1}$ is the equilibrium transition function that maps current states into next period's states. Notice that $\Psi(i, \omega_t) = L(\omega_t, \Theta(i, \omega_t))$ and $\Psi^*(\omega_t) = L(\omega_t, \Theta^*(\omega_t))$. To be clear which argument the partial derivatives refer to, write $L_t \equiv L(\omega_t, \omega_{t+1})$. Differentiating with respect to ω_t gives

$$\begin{aligned} D_{\omega_t}\Psi^* \;&=\; D_{\omega_t}L_t \;+\; D_{\omega_t}\Theta^* \;\cdot\; D_{\omega_{t+1}}L_t \\ &=\; -D_{\omega_t}Q \cdot [D_{a_t}Q]^{-1} \;+\; D_{\omega_t}\Theta^* \;\cdot\; D_{\omega_{t+1}}L_t \end{aligned} \tag{23}$$

Iterating (23) one period forward, substituting it into (21), and then substituting the resulting expression into the first order condition (14) of date t leader i_t, yields the following *Distortion-adjusted Euler equation*

$$D_{a_t}u_{i_t} \;+\; \delta\; D_{a_t}Q \;\cdot\; [R(i_t, \omega_{t+1};\; \Psi^*) \;+\; P(i_t, \omega_{t+1};\; \Psi^*)] \;\;=\;\; 0 \tag{24}$$

where $\omega_{t+1} = Q(\omega_t, \Psi^*(\omega_t))$, and

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[19] A real valued function f is *supermodular in the vector* (x, y) if for all (x, y),

$$f(x \vee y) - f(x) \leq f(y) - f(x \wedge y).$$

[20] It is typically used to establish monotonicity in the state. See Roberts (1998,1999) for a similar application of monotone comparative statics to a related dynamic political economy model. He studies a dynamic game of club admissions in which the pivotal voter in the club at date t admits a new club member in $t{+}1$. Attributes of new club members appear directly in the preferences of current voters. A single crossing condition guarantees that process of change is monotone in some well ordered space of member characteristics.

[21] Clearly, if there is decreasing differences in the pair (i, ω), by a simple re-ordering of i, Assumption (A2) can be satisfied.

$$R(i_t, \omega_{t+1};\ \Psi^*) \ \equiv\ D_{\omega_{t+1}} u_{i_t} \ +\ D_{\omega_{t+1}} L_{t+1} \ \cdot\ D_{a_{t+1}} u_{i_t} \qquad (24.a)$$

and

$$P(i_t, \omega_{t+1};\ \Psi^*) \ \equiv\ D_{\omega_{t+1}} \Theta^* \ \cdot\ D_{\omega_{t+2}} L_{t+1} \ \cdot\ \Delta(i_t, \omega_{t+1}; \Psi^*) \qquad (24.b)$$

The Distortion-adjusted Euler Equation illustrates a basic decomposition of motives of any leader when power is policy-endogenous. Each leader weighs the marginal cost $D_{a_t} u_{i_t}$ against two types of marginal gains/losses. One such effect is obvious. A marginal effect on i_t's payoff is brought about when the policy induces a different, and clearly less desirable, political authority in the future. This effect is given above by $P(i_t, \omega_{t+1}; \Psi^*)$. We will refer to it as the *Preservation Effect*. Intuitively, the preservation effect induces the current leader to choose "more conservatively" than under permanent authority, in the sense that it induces him to choose policies that decrease the rate of political change. We show this more precisely below.

The preservation effect vanishes under permanent authority due to the Envelope Theorem. The Euler equation with i_0 as the permanent authority, is given by

$$D_{a_t} u_{i_0} \ +\ \delta\ D_{a_t} Q \ \cdot\ R(i_0, \omega_{t+1}; \psi) \ =\ 0 \qquad (25)$$

where $\omega_{t+1} = Q(\omega_t, \psi(\omega_t))$ and $R(i_0, \omega_{t+1}; \cdot)$ is defined by (24.a) and is evaluated by the PA equilibrium ψ rather than by Ψ^*.

For an arbitrary citizen-type i, the function $R(i, \omega_{t+1}; \Psi^*)$ produces a marginal outcome which is distinct from the preservation effect. R produces both a direct gain in i's payoff next period and an indirect gain from increased productivity of future policies. This effect is present, though differing in magnitude, in both the PA and PE regimes, and is standard in Euler equations. Using the Example of the previous section for intuition, when power shifts from lower to higher marginal valuation types, the change represents a commitment to more progressive policies than would be the case if the leader himself were making the same decisions in those future dates. The upshot is that in the Example, the leader can count on more progressive policies in the future, thus the marginal productivity of the present policy is higher. This intuition is, in fact, true generally as indicated by the Lemma.

Lemma 1 *Suppose Assumptions (A1) and (A2) hold. For each citizen-type* i *and each state* ω_{t+1}, $R(i, \omega_{t+1}; \Psi^*) > R(i, \omega_{t+1}; \psi)$ *if and only if* $\Psi^*(\omega_{t+1}) > \psi(\omega_{t+1})$.

We refer to the difference $R(i, \omega_{t+1}; \Psi^*) - R(i, \omega_{t+1}; \psi)$ as the *Reformation Effect* because it reflects the distortion in marginal payoffs that is due to changes in the state created by the changes in policy as political power shifts. This distortion leads to a distortion in incentives of the particular decision maker. Thus, according to the Lemma, the reformation effect is positive whenever the PE equilibrium produces a more aggressive policy decision than in the PA equilibrium.

The net effect on continuation payoffs of policy-endogenous power relative to permanent power for an arbitrary type i is

$$\overbrace{R(i,\omega_{t+1};\ \Psi^*)\ -\ R(i,\omega_{t+1};\psi)}^{\text{Reformation Effect}}\ +\ \overbrace{P(i,\omega_{t+1};\ \Psi^*)}^{\text{Preservation Effect}} \tag{26}$$

If one can show that (26) is positive state-by-state, then a citizen i chooses a higher policy in the hypothetical PE equilibrium than if that same individual retained permanent authority.

4.2 Monotonicity Results

Theorem 1 *Suppose (A1) and (A2) hold. Let* Ψ^* *be a smooth-limit PE equilibrium and let* Θ^* *be the associated PE equilibrium transition rule. Then,* Θ^* *is weakly increasing in* ω_t.

The Theorem shows that the PE equilibrium transition rule is monotone in states. By itself, this does not say much about Faustian dynamics. However, under a simple initial condition, the following Corollary asserts that Faustian dynamics are monotone.

Corollary *Suppose that, in addition to the assumptions of Theorem 1,* $\Theta^*(\omega_0) > \omega_0$. *Then the PE equilibrium paths of states* $\{\omega_t\}$ *and leaders* $\{i_t\}$ *are weakly increasing:* $\omega_{t+1} \geq \omega_t$ *and* $i_{t+1} \geq i_t$.

The Corollary asserts that the PE equilibrium path of states and decision-making types is increasing provided it starts off that way. Hence, current leaders knowingly lose power to more progressive decision types, and this evolution continues until either a steady state is reached, or until the largest (most progressive) type acquires power. The following result sharpens the characterization.

Theorem 2 *Suppose (A1) - (A2) hold. Let* Ψ^* *be a smooth-limit PE equilibrium. For each state* ω_{t+1}, $P(i,\omega_{t+1};\ \Psi^*)$ *is increasing in* i, *and*

$$P(i,\omega_{t+1};\ \Psi^*)\ <\ 0 \quad \textit{if and only if} \quad i < \mu(\omega_{t+1}) \tag{27}$$

Looking at incentives under the preservation effect in isolation, the result implies that political actors who hold power act in such a way as to offset the evolution of political authority. Hence if evolution is toward more progressive leaders, then the current leader chooses a less progressive policy. If the evolution is toward less progressive leaders, then the current leader acts more progressively. Combining Theorem 2 with the initial condition that $\Theta^*(\omega_0) > \omega_0$ implies that the political preservation effect is *negative.*

Theorem 3 *Suppose (A1) and (A2) hold. Let* Ψ^* *and* ψ *be smooth-limit PE and PA equilibria, respectively. Then, for any state* $\omega_t \geq \omega_0$, $\Psi^*(\omega_t) \geq \Psi(i_0, \omega_t)$ *with strict inequality if* $\omega_t > \omega_0$.

In words, the hypothetical PE rule is more conservative (less progressive) in every state than in the PE equilibrium. Intuitively, the initial leader i_0 knows that his most preferred policy choice (which he would choose in the PA equilibrium) places power in the hands of more progressive types in the future. Knowing this, he must weigh the preservation effect which leads him to choose more conservatively, against the reservation effect which leads him to choose more progressively due to the policy-complementary between current and future policies. On balance his hypothetical policy decision is more conservative than that of the actual leader in equilibrium. See the Appendix for details.

5 Summary, Related Literature, and Extensions

This paper analyzes the dynamics of a classic "Faustian trade-off" between policy and political power. We characterize this trade-off in terms of a distortion-adjusted Euler equation that illustrates the two main motives of a political actor: the desire for self preservation on the one hand, and the need for policy reformation on the other. We demonstrate how the tradeoff works in both a parametric special case, and in a general monotone model. Our results focus on the role of political bias in creating the Faustian trade-off; we show that more biased institutions generally give rise a faster, and sometimes uneven, evolution of political power.

The parametric results show that decision makers can be individually more conservative (in the traditional sense of slowing the process of political change) than if their power were permanent. There are many examples of this. It is often suggested that electoral pressures in the U.S., for instance, induce politicians to adopt a more anti-immigration stances than they otherwise might take.

> "Rep. Ileana Ros-Lehtinen has pleaded with Republican colleagues to consider an immigration proposal that doesn't stop at building a fence on the U.S. border. But the Florida lawmaker said this week House members remain convinced that backing a path to legal status for illegal immigrants would be political suicide. 'The future electoral landscape is of little to no concern to our members,' Ros-Lehtinen said of her fellow Republicans. 'They're not worried about mañana (tomorrow).

> It's all about hoy (today).' " (L. Clark, reported in *The Seattle Times*, June 9, 2007.)

While leaders are individually conservative, we show that their decisions do not halt the transfer of power along the PE equilibrium path (which, if true for the case of U.S. immigration policy, would imply that the long-run dynamics will favor greater immigration.)

The juxtaposition between individual incentives and realized equilibrium behavior is driven in part by a dynamic inconsistency between current and future decision makers that is vaguely similar to the much-studied hyperbolic $\beta - \delta$ policy models.[22] The PE environment is more complicated since while the conflict in the hyperbolic model is exogenously determined by the fixed $\beta - \delta$ parameters, the degree of conflict in the current model is endogenous. At each decision date, the policy determines the degree of conflict between the current and next decision date.

Interestingly, our model can be extended to capture both the exogenous and endogenous forms of dynamic inconsistency. Introducing a nonstationary authority function, $\mu(\omega, t)$, one can then decompose the preservation effect further into "institutional" and "non-institutional" parts. The non-institutional part arises in all dynamically inconsistent models (e.g., the hyperbolic preferences) due solely to the conflict between current and future decision makers as calendar time changes the identity of the decision maker. The pure "Faustian" distortion would then be that part of $P\left(i, \omega_{t+1}; \Psi^*\right)$ that isolates the marginal cost of a change in leadership that works through the state variable, ω. A model of Azzimonti (2005) has related type of decomposition — her paper is discussed in more detail below.

A number of distinct mechanisms for policy-endogenous change have been studied in the literature. Hassler, et. al. (2003) investigate the evolution of the welfare state in a parametric overlapping-generations model. A majority vote determines the level of transfers to unsuccessful agents. Because the population sizes of different types are endogenously determined by individual investment decisions, their model can generate a shift of political power even with majority voting.[23]

Campante (2007) examines a mechanism through which campaign contributions have policy-endogenous effects. He provides evidence from U.S. elections to support the claim that political process exhibits a wealth bias.

Ortega (2005) studies a natural policy-endogenous mechanism in the form of an immigration quota. Each period, a majority vote by current citizens determines which types of immigrants to let in. Ceteris paribus, current residents want to admit immigrants with com-

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[22] See Laibson (1997), Harris and Laibson (2001), Krusell and Smith (2002), Krusell, Kuruscu and Smith (2002), Judd (2004)).

[23] Although, in their model, endogenous change in political power occurs mainly through private sector investment decisions rather than directly from current policies. A related model and policy-endogenous mechanism is studied in Hassler, et. al. (2005).

plementary skills. On the other hand such immigrants are future voters who will vote to admit future immigrants whose skills are substitutes to those of the current residents. This trade-off can be viewed as a particular instance of the preservation and reformation effects we identify.

Azzimonti (2005) posits an interesting model of dynamic inefficiency in government. The inefficiency arises because a dominant faction loses power to another due to political shocks. Initially, the shocks are exogenous, and so power is not policy-endogenous. However, she later endogenizes the switching likelihoods between the two factions by introducing probabilistic voting. When the shocks to voters' ideological preferences for one group are asymmetric, then increases in public spending change voters' relative preferences between the groups, and so the identity of the pivotal voter changes as well. Significantly, Azzimonti emphasizes the decomposition of motives in an Euler equation related to the one in the parametric model.[24]

Our general model is complementary to these studies in the sense that it highlights what we view as representative attributes of the policy-power trade-off that exist in a wide spectrum of political institutions and policy objectives. As it turns out, many of the insights of these papers, and of Azzimonti's Euler decomposition, for instance, as well as those of the parametric example in this paper, generalize quite broadly to a large class of smooth policy-endogenous models.

A somewhat different literature allows for policy-endogenous power, but also allows the participants to undo its effects by un-coupling the policy and institutional decisions. For instance, Acemoglu and Robinson (2000, 2001, 2005), Cervelatti, et. al. (2006), Jack and Lagunoff (2006a,b), and Lagunoff (2006a) all examine models of explicit institutional (de jure) choices by current rulers, as a way of reversing or mitigating the deleterious effects of current policy on one's future political fortunes.

Similarly, Person and Tabellini (2007) examine the nexus of political and economic capital. They build a model in which economic and political investments are mutually reinforcing in democracies. The success of an attempted coup depends on the political investments of the citizenry. Because democracies generate higher returns to economic investments, its citizens are willing to invest more political capital to defend it against coups.

In a related vein, Roberts (1998, 1999) examines a model of political change in which a pivotal voter in period t directly chooses a pivotal voter at date $t + 1$. Similarly, Barbera, Maschler, and Shalev (BMS) (2001) consider a model related to Ortega's in which current members of the club vote on "who to let into the club" in the future. In both Robert's and BMS' work, the attributes of a future voter appear directly in the preferences of the current voter. In these models the policy itself *is* the composition of political power.

Another paper by Acemoglu and Robinson, (2006), explores this "un-coupling" idea explicitly. They examine persistence of *de facto* power in the face of institutional change. Building on

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[24] Interestingly, her decomposition also includes exogenous inconsistency, a lá hyperbolic decision making, arising due to shocks rather than due to non-stationarity.

an earlier framework laid out in Acemoglu, Johnson, and Robinson (2005), they model the difference between *de facto* and *de jure* power explicitly by examining the economic mechanisms through which the elites can prevent or undo the steps taken by a country to democratize. They identify "captured democracies" as those in which the elite's investments succeed in preserving its de facto power despite the democratizing de jure changes in institutions.

The key difference between their model and ours is that in our model, policies generate political change, while in theirs, policies are used by elites to undo political change. Thus, our mechanism is, in a sense, the reverse of theirs. Clearly, there are examples of both. Acemoglu and Robinson look to 20th century Latin America for numerous instances of captured democracies. On the other hand, the collapse of the Soviet Union, and the role of Glasnost in facilitating the change in power, is an instance where a decision maker (Gorbachev) made hard choices leading to a, perhaps unavoidable, loss of his own power.

A few modeling choices warrant further discussion. First, decision makers are assumed to be exclusively policy-driven. Their desire for power is therefore purely instrumental. (This is in keeping with the original depiction of Faust as a well-intentioned character.) There are likely many historical examples of leaders who desire power for its own sake. It would not be difficult to incorporate "power-hungry" leaders into the model, however, this extension would be, in our view, rather prosaic.

Second, we omit stochastic shocks. Decision makers in the model choose not so much *whether* to lose power, but by *how much* and *to whom*. Consequently, we omit the case where leaders are uncertain about the political ramifications of their policies. As it turns out shocks do not fundamentally change the nature of the Faustian trade-off. They do introduce, however, risk aversion into the motives of the leader, and for this reason, would be a useful addition to future work.

Third, throughout the analysis, we keep the political institution exogenous in the analysis. We focus only on the de facto evolution of the political power within a stable (*de jure*) political institution. This allows us to examine the consequences of exogenous changes in political institutions. By construction, the framework does not answer the question of why a certain political institution is chosen and what determines the evolution of the de jure political institution. Future work could investigate the interaction of the policy-endogenous political power and policy-endogenous political institutions.

Finally, we do not explore in much detail how explicit political mechanisms map into authority. We have a number of results on this, and a related paper (in progress) examines more fully the role economic fundamentals in determining these mechanisms.

6 Appendix

Proof of Parts (i) in Propositions 1 and 2. For brevity, we combine Parts (i) of Propositions 1 and 2, since the argument does not depend per se on whether $\kappa > 0$ or $\kappa < 0$.

We first conjecture a solution Ψ^* of the form $\Psi^*(\omega) = A^* + B^*\omega$. The conjecture is used to characterize both Ψ^* and Ψ, the hypothetical rule. We establish a solution for coefficients and establish uniqueness. We then show $\kappa = 0$, and show that $B^* = 0$ in this case. This case gives us the PA equilibrium ψ Finally, we solve for the steady state $\bar{\omega}$ of the PE equilibrium.

Step 1°. *Verifying the Functional Forms.* We verify that a unique PE equilibrium exists with the affine form $\Psi^*(\omega) = (d - K)\bar{\omega} + K\omega$ where K is a constant (in ω), though we later show how it varies with κ. $\bar{\omega}$ is the unique steady state which depends on κ and κ_0. In this formulation, $A^* = (d - K)\bar{\omega}$ and $B^* = K$. Using this affine form as our "guess" the flow utility is

$$\begin{aligned} u\left(i,\omega,a\right) &= f\left(i\right)\omega - \frac{1}{2}a^2 \\ &= f\left(i\right)\omega - \frac{1}{2}\left(K\left(\omega - \overline{\omega}\right) + d\bar{\omega}\right)^2 \\ &= f\left(i\right)\bar{\omega} - \frac{1}{2}d^2\bar{\omega}^2 + \left(f\left(i\right) - d\bar{\omega}K\right)\left(\omega - \bar{\omega}\right) - \frac{1}{2}K^2\left(\omega - \overline{\omega}\right)^2 \end{aligned}$$

For the purpose of solving for the equilibrium, we can drop the constant term. The continuation utility for an arbitrary i is

$$\begin{aligned} V\left(i,\omega_t;\ \Psi^*\right) &= \sum_{s=0}^{\infty}\delta^s u\left(i,\omega_{t+s},|psi^*(\omega_{t+s})\right), \\ &= \sum_{s=0}^{\infty}\delta^s\left[\left(f\left(i\right) - d\overline{\omega}K\right)\left(\omega_{t+s} - \bar{\omega}\right) - \frac{1}{2}K^2\left(\omega_{t+s} - \bar{\omega}\right)^2\right] \\ &= \frac{f\left(i\right) - d\bar{\omega}K}{1 - \delta\left(K + 1 - d\right)}\left(\omega_t - \bar{\omega}\right) - \frac{1}{2}\frac{K^2}{1 - \delta\left(K + 1 - d\right)^2}\left(\omega_t - \overline{\omega}\right)^2. \end{aligned}$$

The last equality above requires convergence of the infinite sum which, in turn, requires $K + 1 - d < \frac{1}{\delta}$ and $\left(K + 1 - d\right)^2 < \frac{1}{\delta}$ which combines to $K + 1 - d < \frac{1}{\sqrt{\delta}}$. The hypothetical problem confronting an arbitrary citizen-type i is

$$\max_{a_t}\ \left\{u\left(i,\omega_t,a_t\right) + \delta V\left(i,\left(1 - d\right)\omega_t + a_t;\ \Psi^*\right)\right\}$$

which, when evaluated at the parametric assumptions produces the first-order condition

$$-a_t + \delta\left[\frac{f\left(i\right) - d\bar{\omega}K}{1 - \delta\left(K + 1 - d\right)} - \frac{K^2}{1 - \delta\left(K + 1 - d\right)^2}\left(\left(1 - d\right)\omega_t + a_t - \bar{\omega}\right)\right] = 0 \tag{28}$$

The first-order condition (28) determines the hypothetical PE policy rule:

$$\begin{aligned}\Psi\left(i,\omega_t\right) &= \frac{1}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}\left[\delta\frac{f\left(i\right)}{1-\delta\left(K+1-d\right)}\right.\\ &\quad\left.+\delta\bar{\omega}K\left(\frac{K}{1-\delta\left(K+1-d\right)^2}-\frac{d}{1-\delta\left(K+1-d\right)}\right)-\frac{\delta K^2\left(1-d\right)}{1-\delta\left(K+1-d\right)^2}\omega_t\right]\end{aligned} \tag{29}$$

We return to the hypothetical equilibrium later. To determine the PE equilibrium, substitute $f\left(i\right)=\kappa_0+\kappa\omega$ in (29) to derive

$$\begin{aligned}\Psi^*\left(\omega_t\right) &= \frac{1}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}\left[\frac{\delta\kappa_0}{1-\delta\left(K+1-d\right)}+\delta\bar{\omega}K\left(\frac{K}{1-\delta\left(K+1-d\right)^2}-\frac{d}{1-\delta\left(K+1-d\right)}\right)\right.\\ &\quad\left.+\left(\frac{\delta\kappa}{1-\delta\left(K+1-d\right)}-\frac{\delta K^2\left(1-d\right)}{1-\delta\left(K+1-d\right)^2}\right)\omega_t\right]\end{aligned} \tag{30}$$

Hence, in order for $\Psi^*(\omega_t)=(d-K)\bar{\omega}+K\omega_t$ to be a PE equilibrium, we must have

$$K = \frac{\frac{\delta\kappa}{1-\delta(K+1-d)}-\frac{\delta K^2(1-d)}{1-\delta(K+1-d)^2}}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}},\quad\text{and} \tag{31}$$

$$\left(d-K\right)\bar{\omega} = \frac{\frac{\delta\kappa_0}{1-\delta(K+1-d)}+\delta\bar{\omega}K\left(\frac{K}{1-\delta(K+1-d)^2}-\frac{d}{1-\delta(K+1-d)}\right)}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}. \tag{32}$$

We therefore have two equations, and two unknowns, K and $\bar{\omega}$. In what follows, we verify that there exists a unique pair $(K,\bar{\omega})$ that satisfy (31) and (32).

Step 2°. Existence and uniqueness of the pair $(K,\bar{\omega})$. The equation for K in (31) can be expressed as

$$K+\frac{\delta K^2\left(K+1-d\right)}{1-\delta\left(K+1-d\right)^2}-\frac{\delta\kappa}{1-\delta\left(K+1-d\right)}=0. \tag{33}$$

Define or $\widehat{K}=K+1-d$. Multiply both sides of the equation (33) by

$$\left(1-\delta\left(K+1-d\right)^2\right)\left(1-\delta\left(K+1-d\right)\right)$$

and after some messy algebra, the equation (33) becomes

$$\begin{aligned}F(\widehat{K}) &\equiv \delta\left(1-d\right)\widehat{K}^3+\left(\kappa\delta-\left(2-d\right)-\delta\left(1-d\right)^2\right)\widehat{K}^2\\ &\quad+\left(\frac{1}{\delta}+\left(1-d\right)+\left(1-d\right)^2\right)\widehat{K}-\left(\kappa+\frac{1}{\delta}\left(1-d\right)\right) = 0.\end{aligned} \tag{34}$$

Observe that the function F is of the form $F(\widehat{K}) = a_0(\kappa) + a_1\widehat{K} + a_2(\kappa)\widehat{K}^2 + a_3\widehat{K}^3$ (given the expression above, it should be clear that a_0 and a_2 vary with κ whereas a_1 and a_3 do not). We then have

$$\begin{aligned}
F\left(1-d\right) &= -\kappa\left(1-\delta\left(1-d\right)^2\right) < 0, \\
F\left(1\right) &= \left(1-\delta\right)\left(d(\frac{1}{\delta}-1+d)-\kappa\right) > 0 \\
F\left(-1\right) &= \left(\kappa+\frac{1-d}{\delta}\right)\left(\delta-1\right)-\left(1+\delta\right)\left[\left(1-d\right)^2+\frac{3+\delta}{1+\delta}\left(1-d\right)+\frac{1}{\delta}\right] \\
F\left(0\right) &= -\left(\kappa+\frac{1}{\delta}\left(1-d\right)\right)
\end{aligned}$$

Suppose first that $0 < \kappa < d\left(\frac{1}{\delta}-1+d\right)$ as required in Proposition 1. Then $F\left(1-d\right) < 0$ and $F\left(1\right) > 0$. Then from the Intermediate Value Theorem, there exists a $\widehat{K}^*$ such that $1-d < \widehat{K}^* < 1$ (or equivalently $0 < K^* < d$) and $F(\widehat{K}^*) = 0$.

Suppose next that $0 > \kappa > -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta}\left[\left(1-d\right)^2 + \frac{3+\delta}{1+\delta}\left(1-d\right)+\frac{1}{\delta}\right]$ as required in Proposition 2. Then $F\left(-1\right) < 0$ and $F\left(1-d\right) > 0$. Once again, we apply the Intermediate Value Theorem to show that there exists a $-1 < \widehat{K}^* < (1-d)$ (or equivalently $-(2-d) < K^* < 0$) such that $F(\widehat{K}^*) = 0$.

To show uniqueness of the solution $\widehat{K}^*$ in either the case of $\kappa > 0$ or $\kappa < 0$, it suffices to show that $F\left(\widehat{K}\right)$ is concave, i.e., $F''\left(\widehat{K}\right) = 2\left(a_2(\kappa)+3a_3\widehat{K}\right) < 0$ for $1-d \le \widehat{K} \le 1$. Towards this goal, it suffices to show that $a_2(\kappa)+3a_3 < 0$ (since $a_3 > 0$), i.e., $\left[\kappa\delta-(2-d)-\delta\left(1-d\right)^2\right] + 3\delta\left(1-d\right) < 0$. The latter is equivalent to the equation

$$\kappa \le d\left(\frac{1}{\delta}-1+d\right)+2\left(\frac{1}{\delta}-1\right)\left(1-d\right)$$

which is always true since $\kappa < d\left(\frac{1}{\delta}-1+d\right)$. We conclude that $\widehat{K}^*$, and hence $K^* = \widehat{K}^*-1+d$, is unique.

Having established a unique solution, K^*, we now solve for the steady state, $\bar{\omega}$. The solution for $\bar{\omega}$ can be solved in the equation

$$\left(d-K\right)\bar{\omega} = \frac{\frac{\delta\kappa_0}{1-\delta(K+1-d)}+\delta\bar{\omega}K\left(\frac{K}{1-\delta(K+1-d)^2}-\frac{d}{1-\delta(K+1-d)}\right)}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}$$

The equation implies that

$$\left[\left(d-K\right)-\frac{\delta K^2\left(K+1-d\right)}{1-\delta\left(K+1-d\right)^2}+\frac{\delta dK}{1-\delta\left(K+1-d\right)}\right]\bar{\omega} = \frac{\delta\kappa_0}{1-\delta\left(K+1-d\right)}$$

Using equation (33), i.e., $K + \frac{\delta K^2(K+1-d)}{1-\delta(K+1-d)^2} = \frac{\delta\kappa}{1-\delta(K+1-d)}$, and some algebra, we obtain

$$\bar{\omega} = \frac{\kappa_0}{d\left(\frac{1}{\delta} - (1-d)\right) - \kappa}, \tag{35}$$

We have therefore established a unique pair $(K^*, \bar{\omega})$ with K^* as the slope of Ψ^* and $\bar{\omega}$ as the steady state satisfying (35). As the solution to $F(K+1-d) = 0$, notice that K^* varies with κ. We write $K^* = B^*(\kappa)$ to emphasize the dependence on κ. Using our newly found equation for $\bar{\omega}$, the constant term for Ψ^* is given by $A^* = (d - B^*(\kappa))\left(\frac{\kappa_0}{d\left(\frac{1}{\delta}-(1-d)\right)-\kappa}\right)$. The coefficients A^* and $B^*(\kappa)$ are those that define the PE equilibrium. By the definition of F in (34), it is clear that $B^*(0) = 0$, which then yields an equation for the PA equilibrium ψ. As for the hypothetical PE rule, Ψ, its solution form is also affine. The coefficients can be recovered K and $\bar{\omega}$ evaluated at their respective solutions. The slope is negative.

Rest of the proof of Proposition 1. We now turn to Part (ii) of Proposition 1. To prove that convergence to the steady state is monotone, observe that

$$\Theta^*(\omega_t) \equiv (1-d)\omega_t + \Psi^*(\omega_t) = (1-d+K^*)\omega_t + (d-K^*)\bar{\omega} \tag{36}$$

Since $0 < K^* < d$ if $0 < \kappa < d\left(\frac{1}{\delta} - 1 + d\right)$, then convergence toward the steady state is monotonically increasing if $\omega_0 < \bar{\omega}$, and monotonically decreasing if $\omega_0 > \bar{\omega}$.

The rest of Part (ii) can be readily verified from a direct comparison of the solutions to ψ and $\Psi(i_0, \omega)$, both of which are constructed from the PE equilibrium above. We omit the details. Part (iii) is a special case of Proposition 3, and so refer the Reader to that proof.

Proof Proposition 2. Part (i) is already proved above. As for Part (ii), an inspection of (36) reveals that $D_\omega\Theta^* > 0$ iff $\widehat{K} = 1 - d + K^* > 0$. Recalling the definition of F in the proof of Proposition 1, it is clear that $F(0) < 0$ and $F(1-d) > 0$ if $0 > \kappa > -(1-d)/\delta$, and so the fixed point $\widehat{K}^*$ of F must satisfy $\widehat{K}^* > 0$. On the other hand, if $-(1-d)/\delta > \kappa > -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta}\left[(1-d)^2 + \frac{3+\delta}{1+\delta}(1-d) + \frac{1}{\delta}\right]$, then $F(0) > 0$ and $F(-1) < 0$, and so the fixed point $\widehat{K}^*$ of F must satisfy $\widehat{K}^* < 0$. In the latter case, the Faustian dynamics constitute a dampened cycle whereby, on-path, $\omega_t < \bar{\omega}$ iff $\omega_{t+1} > \bar{\omega}$. The rest of the proof is a special case of Proposition 3.

Proof Proposition 3. Note that Part (iii) of Proposition 1 is a special case of this result. To prove Proposition 3, we will need to further characterize properties of the solution $K^* = B^*(\kappa)$. But since K^* and $\widehat{K}^*$ differ only by a constant, define $\widehat{B}^*(\kappa) \equiv B^*(\kappa) + 1 - d$. Without loss of generality, we examine properties of the solution $\widehat{K}^* = \widehat{B}^*(\kappa)$ below. The unique solution $\widehat{K}^* = \widehat{B}^*(\kappa)$ is defined implicitly by the equation $F\left(\widehat{K}\right) = 0$. Notice that, at the solution $\widehat{K}^{*}$, $F\left(\widehat{K}^{*}\right)$ satisfies

$$\begin{aligned} F'\left(\widehat{K}^{*}\right) &= a_1 + 2a_2(\kappa)\widehat{K}^{*} + 3a_3\left(\widehat{K}^{*}\right)^2 > 0, \\ F''\left(\widehat{K}^{*}\right) &= 2\left(a_2(\kappa) + 3a_3\widehat{K}^{*}\right) < 0. \end{aligned}$$

Since $F'\left(\widehat{B}^{*}(\kappa)\right) > 0$, $\widehat{B}^{*}(\kappa)$ is continuously differentiable from Implicit Function Theorem. In addition, we know that

$$\widehat{B}^{*\prime}(\kappa) = \frac{1-\delta\left(\widehat{B}^{*}(\kappa)\right)^2}{a_1 + 2a_2(\kappa)\widehat{B}^{*}(\kappa) + 3a_3\left(\widehat{B}^{*}(\kappa)\right)^2}.$$

Take derivative again and after some algebra, we have

$$\widehat{B}^{*\prime\prime}(\kappa) = \frac{-2\left(1-\delta\left(\widehat{B}^{*}(\kappa)\right)^2\right)\left[3\delta a_3\left(\widehat{B}^{*}(\kappa)\right)^3 + 3\delta a_2(\kappa)\left(\widehat{B}^{*}(\kappa)\right)^2 + (2\delta a_1 + 3a_3)\widehat{B}^{*}(\kappa) + a_2(\kappa)\right]}{\left[a_1 + 2a_2(\kappa)\widehat{B}^{*}(\kappa) + 3a_3\left(\widehat{B}^{*}(\kappa)\right)^2\right]^3}$$

Since $a_0(\kappa) + a_1\widehat{B}^{*}(\kappa) + a_2(\kappa)\left(\widehat{B}^{*}(\kappa)\right)^2 + a_3\left(\widehat{B}^{*}(\kappa)\right)^3 = 0$, we know that $a_2(\kappa)\left(\widehat{B}^{*}(\kappa)\right)^2 + a_3\left(\widehat{B}^{*}(\kappa)\right)^3 = -a_0 - a_1\widehat{B}^{*}(\kappa)$. As a result, we have

$$\begin{aligned} & 3\delta a_3\left(\widehat{B}^{*}(\kappa)\right)^3 + 3\delta a_2(\kappa)\left(\widehat{B}^{*}(\kappa)\right)^2 + (2\delta a_1 + 3a_3)\widehat{B}^{*}(\kappa) + a_2(\kappa) \\ = & \ 3\delta\left(-a_0 - a_1\widehat{B}^{*}(\kappa)\right) + (2\delta a_1 + 3a_3)\widehat{B}^{*}(\kappa) + a_2(\kappa) \\ = & -\left(1-\delta\left(1-d^2\right)\right)\left[\widehat{B}^{*}(\kappa) - \frac{-\delta d^2 - 2(1-\delta)d + (1-\delta) + 4\delta\kappa}{1-\delta(1-d^2)}\right]. \end{aligned}$$

To summarize, we have

$$\widehat{B}^{*\prime\prime}(\kappa) = \frac{2\left(1-\delta\left(\widehat{B}^{*}(\kappa)\right)^2\right)\left(1-\delta\left(1-d^2\right)\right)\left[\widehat{B}^{*}(\kappa) - b(\kappa)\right]}{\left[a_1 + 2a_2(\kappa)\widehat{B}^{*}(\kappa) + 3a_3\left(\widehat{B}^{*}(\kappa)\right)^2\right]^3},$$

where $b(\kappa) = \frac{-\delta d^2 - 2(1-\delta)d + (1-\delta) + 4\delta\kappa}{1-\delta(1-d^2)}$. Consequently, the sign of $\widehat{B}^{*\prime\prime}(\kappa)$ is the same as that of $\widehat{B}^{*}(\kappa) - b(\kappa)$.

In the following part, we show that there exists a $\overline{\kappa}$ such that (i) $\widehat{B}^*(\kappa) > b(\kappa)$ for $\kappa < \overline{\kappa}$; (ii) $\widehat{B}^*(\overline{\kappa}) = b(\overline{\kappa})$ for $\kappa = \overline{\kappa}$; (iii) $\widehat{B}^*(\kappa) < b(\kappa)$ for $\kappa > \overline{\kappa}$. To start with, we show that

$$\begin{aligned}
\widehat{B}^*(0) > b(0) \iff & \; 1 - d > \frac{-\delta d^2 - 2(1-\delta)d + (1-\delta)}{1 - \delta(1-d^2)} \\
\iff & \; 1 - \delta(1-d)^2 \; > \; 0,
\end{aligned}$$

and

$$\begin{aligned}
& \widehat{B}^*\left(d\left(\frac{1}{\delta} - 1 + d\right)\right) \; \leq \; b\left(d\left(\frac{1}{\delta} - 1 + d\right)\right) \\
\iff & \; 1 \; \leq \; \frac{-\delta d^2 - 2(1-\delta)d + (1-\delta) + 4\delta d\left(\frac{1}{\delta} - 1 + d\right)}{1 - \delta(1-d^2)} \\
\iff & \; 0 \; \leq \; 2d(\delta d + 1 - \delta).
\end{aligned}$$

Therefore, from the Intermediate Value Theorem there exists a $\overline{\kappa}$ such that $\widehat{B}^*(\overline{\kappa}) = b(\overline{\kappa})$.

Now we show that $\widehat{B}^*(\kappa) > b(\kappa)$ for $\kappa < \overline{\kappa}$ and $\widehat{B}^*(\kappa) < b(\kappa)$ for $\kappa > \overline{\kappa}$. To show this, we have

$$G(\kappa) = F(b(\kappa)) = a_3(b(\kappa))^3 + a_2(\kappa)(b(\kappa))^2 + a_1 b(\kappa) + a_0(\kappa),$$

and the derivative of $G(\kappa)$ can be calculated as

$$G'(\kappa) = \left[3a_3(b(\kappa))^2 b'(\kappa) + 2a_2(\kappa)b(\kappa)b'(\kappa) + \delta(b(\kappa))^2 + a_1 b'(\kappa) - 1\right]$$

which equals

$$\begin{aligned}
& \frac{6}{1 - \delta(1-d^2)} \left[\frac{1}{2}\delta\left[\delta d^2 - 4\delta d + 1 + 3\delta\right](b(\kappa))^2\right. \\
& \left. - \delta\left[\delta d^2 - 2(1+\delta)d + (3+\delta)\right] b(\kappa) \frac{1}{2}\left[\delta d^2 - 4\delta d + 1 + 3\delta\right]\right]
\end{aligned}$$

To see the sign of $G'(\kappa)$, first notice that

$$\begin{aligned}
& \delta^2\left[\delta d^2 - 2(1+\delta)d + (3+\delta)\right]^2 - \delta\left[\delta d^2 - 4\delta d + 1 + 3\delta\right]^2 \\
= \; & \delta^2\left[1 - \delta(1-d)^2 - 2(2-d)\right]^2 - \delta\left[1 - \delta(1-d)^2 - 2(1 + \delta(1-d))\right]^2 \\
= \; & \delta(\delta - 1)\left(1 - \delta(1-d)^2\right) \; < \; 0.
\end{aligned}$$

Combine this with the fact that $\delta d^2 - 4\delta d + 1 + 3\delta = \delta d^2 + (1 - \delta d) + 3\delta(1-d) > 0$, we know that $G'(\kappa) > 0$ for every κ. As a result, $G(\kappa) < 0$ for $\kappa < \overline{\kappa}$ and $G(\kappa) > 0$ for $\kappa > \overline{\kappa}$. Therefore, $\widehat{B}^*(\kappa) > b(\kappa)$ for $\kappa < \overline{\kappa}$ and $\widehat{B}^*(\kappa) < b(\kappa)$ for $\kappa > \overline{\kappa}$.

Finally, we show that $\widehat{B}^{*}(\kappa) > \frac{\kappa}{\frac{1}{\delta}-1+d}, \forall \kappa$. To start with, notice that

$$\begin{aligned} \widehat{B}^{*\prime}(0) &= \frac{1-\delta(1-d)^2}{a_1 + 2a_2(\kappa)(1-d) + 3a_3(1-d)^2}, \\ &= \frac{1}{\frac{1}{\delta}-1+d}, \end{aligned}$$

Combine $\widehat{B}^{*\prime}(0) = \frac{1}{\frac{1}{\delta}-1+d}$ and the property of $\widehat{B}^{*\prime\prime}(\kappa)$, we know that $\widehat{B}^{*}(\kappa) > \frac{\kappa}{\frac{1}{\delta}-1+d}, \forall \kappa$.

Translating back to our original coefficient, $B^{*}(\kappa)$, we have thus established: $B^{*'} > 0$, and $B^{*''}(\kappa) > 0$ for $\kappa < \bar{\kappa}$ and $B^{*''}(\kappa) < 0$ for $\kappa > \bar{\kappa}$.

Now we return directly to the hypothesis of Proposition 3. Let K^{*} and $\tilde{K}^{*}$ denote the respective solutions for PE equilibria Ψ^{*} and $\tilde{\Psi}^{*}$, resp. Using our constructions, we want to show that,

$$\begin{aligned} & \left(d-\widetilde{K}^{*}\right)\frac{\widetilde{\kappa}_0}{d\left(\frac{1}{\delta}-1+d\right)-\widetilde{\kappa}} + \widetilde{K}^{*}\omega_0 > (d-K^{*})\frac{\kappa_0}{d\left(\frac{1}{\delta}-1+d\right)-\kappa} + K^{*}\omega_0 \\ \Longleftrightarrow \quad & \frac{d-\widetilde{K}^{*}}{d\left(\frac{1}{\delta}-1+d\right)-\widetilde{\kappa}} < \frac{K^{*}-\widetilde{K}^{*}}{\kappa-\widetilde{\kappa}}. \end{aligned}$$

But the last inequality follows from the fact $\frac{\kappa}{\frac{1}{\delta}-1+d} < K$ and the properties of $B^{*\prime\prime}(\kappa)$. This concludes the proof of the Proposition. ▮

Proof of Lemma 1 To prove the result, we first the establish the following Claim.

Claim 1 *The function L ($a = L(\omega, \omega')$) exhibits* decreasing differences *in ω and ω'.*

Proof of the claim. Cross differentiating the equation $Q(\omega, L(\omega, \omega')) = \omega'$ with respect to ω' and ω gives

$$D_{\omega} D_a Q \cdot D_{\omega}' L \;+\; D_a^2 Q \cdot D_{\omega}' L \cdot D_{\omega} L \;+\; D_a Q \cdot D_{\omega} D_{\omega'} L \;=\; 0 \tag{37}$$

By supermodularity of Q (from Assumption (A2)) and the fact that L is increasing in ω', the first term is positive. The second term is positive due to concavity of Q and our earlier monotonicity results on L. In order to satisfy the equality, we therefore require $D_{\omega} D_{\omega'} L < 0$. Consequently, L exhibits decreasing differences in ω and ω'. We have therefore proved the Claim. ▮

Now, recall from (24.a) that

$$R(i, \omega_t;\, \Psi^{*}) \;\equiv\; D_{\omega_t} u(i, \omega_t, \Psi^{*}(\omega_t)) \;+\; D_{\omega_t} L(\omega, \Theta^{*}(\omega_t)) \;\cdot\; D_{a_t} u(i, \omega_t, \Psi^{*}(\omega_t)) \tag{38}$$

Define $\theta(\omega_t)$ implicitly by $\psi(\omega_t) = L(\omega_t, \theta(\omega_t))$. By extension of the definition of R in (24.a) we have

$$R(i, \omega_t;\ \psi) \ \equiv\ D_{\omega_t} u(i, \omega_t, \psi(\omega_t)) \ + \ D_{\omega_t} L(\omega, \theta(\omega_t)) \ \cdot\ D_{a_t} u(i, \omega_t, \psi(\omega_t)) \tag{39}$$

It follows directly from (A2) that $\Psi^*(\omega_t) > \psi(\omega_t)$ holds whenever $D_{\omega_t} u(i, \omega_t, \Psi^*(\omega_t)) > D_{\omega_t} u(i, \omega_t, \psi(\omega_t))$. Also by strict concavity of u,

$$D_{a_t} u(i, \omega_t, \psi(\omega_t)) > D_{a_t} u(i, \omega_t, \Psi^*(\omega_t))$$

whenever $\Psi^*(\omega_t) > \psi(\omega_t)$. But since $D_{a_t} u < 0$ by monotonicity of u in the policy, the Lemma is established if we can show both $D_{\omega_t} L < 0$ and $D_{\omega_t} L(\omega, \theta(\omega_t)) > D_{\omega_t} L(\omega, \Theta^*(\omega_t))$ whenever $\psi(\omega_t) < \Psi^*(\omega_t)$.

By definition of $\Psi^*(\omega_t) = L(\omega_t, \Theta^*(\omega_t))$ and $\psi(\omega_t) = L(\omega_t, \theta(\omega_t))$. Consequently, by the monotonicity of L in ω', $\Psi^*(\omega_t) > \psi(\omega_t)$ whenever $\Theta^*(\omega_t) > \theta(\omega_t)$. The application of Claim 1 therefore gives $D_{\omega_t} L < 0$ and $D_{\omega_t} L(\omega, \theta(\omega_t)) > D_{\omega_t} L(\omega, \Theta^*(\omega_t))$. Hence, we have shown

$$\begin{aligned} & D_{\omega_t} u(i, \omega_t, \Psi^*(\omega_t)) \ + \ D_{\omega_t} L(\omega, \Theta^*(\omega_t)) \ \cdot\ D_{a_t} u(i, \omega_t, \Psi^*(\omega_t)) \\ > \ & D_{\omega_t} u(i, \omega_t, \psi(\omega_t)) \ + \ D_{\omega_t} L(\omega, \theta(\omega_t)) \ \cdot\ D_{a_t} u(i, \omega_t, \psi(\omega_t)) \end{aligned} \tag{40}$$

whenever $\Psi^*(\omega_t) > \psi(\omega_t)$. We conclude the proof of the Lemma. ∎

Proof of Theorem 1 For any arbitrary, smooth continuation value $U(i, \omega)$, define

$$H(i, \omega_t, a_t, U) = u(i, \omega_t, a_t) \ + \ \delta\ U(i, Q(\omega_t, a_t)) \tag{41}$$

$H(i, \omega_t, a_t, U)$ is the payoff function of a citizen-type i in state ω_t when his continuation is defined by $U(i, \omega_{t+1})$. Note that when U is the PE equilibrium continuation value, then by definition, $U(i, \omega_{t+1}) = V(i, \omega_{t+1};\ \Psi^*)$.

Given $H(i, \omega_t, a_t, U)$ as defined above, let $\Psi(i, \omega, U) \in \arg\max_a H(i, \omega_t, a_t, U)$, and let $\Psi^*(\omega, U) = \Psi(\mu(\omega), \omega, U)$. Recall that $a_t = L(\omega_t, \omega_{t+1})$ and consider $H(i, \omega_t, L(\omega_t, \omega_{t+1}), U)$.

We now proceed with the proof. We first establish $D_\omega D_{\omega'} H(i, \omega_t, L(\omega, \omega'), U) > 0$. To verify this, notice that the continuation value U does not vary with the current state ω. Hence, to show supermodularity in ω and ω' (we drop the time subscript for convenience), it suffices to show that $u(i, \omega, L(\omega, \omega'))$ is supermodular in (ω, ω'). Totally differentiating u with respect to ω yields

$$D_\omega u + D_a u \cdot D_\omega L$$

Differentiating this expression with respect to ω' gives

$$D_\omega D_{\omega'} u \ = \ D_\omega D_a \cdot D_{\omega'} L \ + \ D_a^2 u \cdot D_{\omega'} L \cdot D_\omega L \ + \ D_a u \cdot D_\omega D_{\omega'} L$$

By supermodularity of u and by Claim 1, the first term is positive. By concavity of u and Claim 1, the second term is positive. Finally, by the fact that u is decreasing in a and once again by Claim 1, the last term is positive. Hence, $D_\omega D_{\omega'} u > 0$ term-by-term, and so H is supermodular in (ω', ω), independently of U. Hence, by Topkis Monotonicity Theorem, $\Theta(i, \omega_t, U)$ is increasing in ω_t for any U. In particular, the hypothetical PE equilibrium transition $\Theta(i, \omega_t)$ is increasing in ω_t.

Next, consider a continuation value U that satisfies $D_i D_\omega U(i, \omega) > 0$. That is, we consider a continuation value with increasing differences in i and ω. We now show, first, that $D_i D_a H > 0$ and second that $D_i D_\omega H(i, \omega, L(\omega, \Theta^*(\omega, U)), U) > 0$. In other words, H has increasing differences in i and a, and when evaluated at Θ^*, H inherits the same increasing difference property as the initial continuation value U. To show these two facts, we take the cross derivatives of the objective function $H(i, \omega, a;\ U)$ in i and a to obtain

$$D_i D_a H \ = \ D_i D_a u \ + \ \delta \ D_a Q \cdot D_i D_{\omega'} U(i, \omega') \ > \ 0$$

This implies that $\Psi(i, \omega, U)$ is increasing in i by the Topkis' Monotonicity Theorem, and so $\Theta(i, \omega, U) = Q(\omega, \Psi(i, \omega, U))$ is also increasing in i. Hence, we have shown that $\Theta^*(\omega, U) = \Theta(\mu(\omega), \omega, U)$ is increasing in ω. Next, consider the cross derivative:

$$\begin{aligned} D_i D_\omega H(i, \omega, L(\omega, \Theta^*(\omega, U)), U) \ &= \ D_i D_\omega u + D_i D_a u \cdot (D_\omega L + D_{\omega'} L \cdot D_\omega \Theta^*) \\ &\quad + \delta D_\omega \Theta^* \cdot D_i D_{\omega'} U(i, \omega') \end{aligned}$$

By Assumption (A2), $D_i D_\omega u + D_i D_a u \cdot D_\omega L > 0$. Since all the remaining terms on the right-hand side are positive, the cross derivative is positive. This implies that the map $U \mapsto H(i, \omega, L(\omega, \Theta^*(\omega, U)), U)$ maps from functions with increasing differences in i and ω to functions of the same. Consider, then the finite horizon PE equilibrium with horizon T. Let $\Theta_T = \{\Theta^*_{t,T}\}_{t=1}^T$ denote the PE equilibrium transition in the T-period model, and let $U_{t,T}$ denote the value function in each period t. Notice that $U_{T,T} = u$ which satisfies increasing differences in i and ω by Assumption (A2). Consequently, $\Theta^*_{T-1,T}$ is increasing in ω.[25] A simple backward induction argument establishes that $U_{t,T}$ satisfies increasing differences and, hence, $\Theta^*_{t,T}$ is increasing in ω for all t. Since the infinite horizon equilibrium Θ^* satisfies $||\Theta^*_{t,T} - \Theta^*|| \to 0$, it follows that Θ^* is increasing in ω_t. Moreover, $||U_{t,T} - V(\cdot\ ;\ \Psi^*)|| \to 0$ as $T \to \infty$, and $V(\cdot\ ;\ \Psi^*)$, the infinite horizon PE equilibrium continuation value, has increasing differences in i and ω. This completes the proof. ∎

Proof of Theorem 2. To simplify notation, let $V^*(i, \omega) \equiv V(i, \omega;\ \Psi^*)$. Using the definition of H in (41) and the inverse function L, one can rewrite the objective function H defined in (41) as

$$H(i, \omega_t, L(\omega_t, \omega_{t+1}),\ V^*) \ = \ u(i, \omega_t, L(\omega_t, \omega_{t+1})) \ + \ \delta V^*(i, \omega_{t+1}) \tag{42}$$

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[25] In the last period T, $\underline{a}$ is chosen by the decision maker since there is no future return to the costly investment.

Using the definition of H in (42) and that of the distortion function Δ in (19), we have

$$\begin{aligned} D_{\omega'_{t+1}} H(i_t, \omega_{t+1}, L(\omega_{t+1}, \Theta^*(\omega_{t+1})),\ V^*) &= D_{a_t} H(i_t, \omega_{t+1}, \Psi^*(\omega_{t+1}), V^*) \cdot D_{\omega'_{t+1}} L(\omega_{t+1}, \Theta^*(\omega_{t+1})) \\ &= \Delta(i_t, \omega_{t+1}; \Psi^*) \cdot D_{\omega'_{t+1}} L(\omega_{t+1}, \Theta^*(\omega_{t+1})) \end{aligned} \tag{43}$$

Using the Proof of Theorem 1, which establishes increasing differences of H in i and ω, it follows that $D_{\omega'_{t+1}} H(i, \omega_{t+1}, L(\omega_{t+1}, \Theta^*(\omega_{t+1}))\ , V^*)$ is increasing in i, and so $\Delta(i, \omega_{t+1}; \Psi^*)$ must be increasing in i_t as well. This proves that $P(i, \omega_t;\ \Psi^*)$ is increasing in i. It also implies that whenever $i < i_{t+1} = \mu(\omega_{t+1})$, then

$$\Delta(i, \omega_{t+1}; \Psi^*)\ <\ \Delta(i_{t+1}, \omega_{t+1}; \Psi^*)\ =\ 0 \tag{44}$$

where the latter equality follows from the type i_{t+1}'s first order condition in state ω_{t+1}. Now recall that the preservation effect is given by

$$P(i, \omega_{t+1};\ \Psi^*)\ \equiv\ D_{\omega'_{t+1}} L\ \cdot\ D_{\omega_{t+1}} \Theta^*\ \cdot\ \Delta(i, \omega_{t+1}; \Psi^*) \tag{45}$$

By Claim 1, $D_{\omega'_{t+1}} L > 0$ and from the proof of Theorem 1, $D_{\omega_{t+1}} \Theta^* > 0$. Therefore, $P(i, \omega_{t+1};\ \Psi^*) < 0$ follows from the fact that $\Delta(i, \omega_{t+1}; \Psi^*) < 0$. ▮

Proof of Theorem 3. The argument in Theorem 1 established $\Theta^*(\omega_t) > \Theta(i_0, \omega_t)$ for all $t > 1$. By definition,

$$\Psi^*(\omega_t) = L(\omega_t, \Theta^*(\omega_t))\ >\ L(\omega_t, \Theta(i_0, \omega_t)) = \Psi(i_0, \omega_t) \tag{46}$$

as required for the Theorem. ▮

References

[1] Acemoglu, D., S. Johnson, and J. Robinson (2005), "Institutions as the Fundamental Cause of Long-Run Growth," *Economic Growth*, P. Aghion and S. Durlauf, eds., North Holland.

[2] Acemoglu, D. and J. Robinson (2000), "Why Did the West Extend the Franchise? Democracy, Inequality and Growth in Historical Perspective," *Quarterly Journal of Economics*, 115: 1167-1199.

[3] Acemoglu, D. and J. Robinson (2001), "A Theory of Political Transitions," *American Economic Review*, 91: 938-963.

[4] Acemoglu, D. and J. Robinson (2006), *Economic Origins of Dictatorship and Democracy*, Cambridge University Press.

[5] Acemoglu, D. and J. Robinson (2007), "Persistence of Powers, Elites and Institutions," *American Economic Review*, forthcoming.

[6] Alesina, A. and D. Rodrik (1994), "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 109: 465 - 490.

[7] Amir, R. (1996), Continuous Stochastic Games of Capital Accumulation with Convex Transitions, *Games and Economic Behavior*, 16: 111-31.

[8] Amador, M. (2003), "A Political Model of Sovereign Debt Repayment," mimeo, Stanford Universty.

[9] Arrow, K. (1951), *Social Choice and Individual Values*, New York: John Wiley and Sons.

[10] Azzimonti, M. (2005), "On the Dynamic Inefficiency of Governments," mimeo, University of Iowa.

[11] Barbera, S., M. Maschler, and S. Shalev (2001),"Voting for voters: A model of electoral evolution," *Games and Economic Behavior*, 37: 40-78.

[12] Athey, S. (1998), "Comparative Statics under Uncertainty: Single Crossing Properties and Log-Supermodularity," mimeo, MIT and NBER.

[13] Basar, J. and Olsder (1982), *Dynamic Non-cooperative Game Theory*, 2nd edition, Academic Press, London/New York.

[14] Battaglini, M. and S. Coate (2007), "Inefficiency in Legislative Policy-Making: A Dynamic Analysis," *American Economic Review*, 97 (1), 118-149.

[15] Bernheim, D. and S. Nataraj (2002), "A Solution Concept for Majority Rule in Dynamic Settings," mimeo, Stanford University.

[16] Bertola, G. (1993), "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 83: 1184 - 1210.

[17] Black, D. (1958), *The Theory of Committees and Elections*, London: Cambridge University Press.

[18] Blackwell (1965), "Discounted Dynamic Programming," *Annals of Mathematical Statistics*, 36:226-35.

[19] Campante, F. (2007), "Redistribution in a Model of Voting and Campaign Contributions," mimeo, Harvard University.

[20] Clark, L (2007), "Even if Senate revives amnesty plan, a tougher challenge looms in House," reported in *The Seattle Times*, June 9, 2007.

[21] Curtat (1996), "Markov Equilibria in Stochastic Games with Complementarities, *Games and Economic Behavior*, 17: 177-99.

[22] Cervellati, M, P. Fortunato, and U. Sunde (2006), "Consensual and Conflictual Democratization," mimeo.

[23] Dahlman, C. (1980), *The Open Field System and Beyond*, Cambridge: Cambridge University Press.

[24] Dutta, P. and R. Sundaram (1994), "The Equilibrium Existence Problem in General Markovian Games," in *Organizations with Incomplete Information: Essays in Economic Analysis, A Tribute to Roy Radner*, M. Majumdar, ed., Cambridge: Cambridge University Press, pp. 159-207.

[25] Egorov G. and K. Sonin (2005), "The Killing Game: Reputation and Knowledge in Non-Democratic Succession," mimeo.

[26] Finer, S.E. (1997), *The History of Government*, Oxford University Press, Oxford, UK.

[27] Gans, J. and M. Smart (1996), "Majority voting with single-crossing preferences," *Journal of Public Economics*, 59: 219-237.

[28] von Goethe, J.W. (19??), *Faust*, English translation by Bayard Taylor, New York: Arden Book Co. (exact publication date unknown). www.openlibrary.org/details/faustgoethe00goetiala.

[29] Gomes, A. and P. Jehiel (2005), "Dynamic Processes of Social and Economic Interactions: On the Persistence of Inefficiencies," *Journal of Political Economy*, 113: 626–667.

[30] Gradstein, M. and M. Justman (1999), "The Industrial Revolution, Political Transition, and the Subsequent Decline in Inequality in 19th Century Britain," *Exploration in Economic History*, 36:109-27

[31] Greif, A. and D. Laitin (2004), "A Theory of Endogenous Institutional Change," *American Political Science Review*, forthcoming.

[32] Harris, C. and D. Laibson (2001), "Dynamic Choices of Hyperbolic Consumers," *Econometrica*, 69, 935-957.

[33] Hassler, J., P. Krusell, K. Storlesletten, and F. Zilibotti (2005), "The Dynamics of Government," *Journal of Monetary Economics* 52: 1331-1358.

[34] Hassler, J., J.V. Rodriguez Mora, K. Storlesletten, and F. Zilibotti (2003), "The Survival of the Welfare State," *American Economic Review* 93: 87-112.

[35] Howard, R. (1960), *Dynamic Programming and Markov Processes*, New York: M.I.T. and John Wiley and Sons.

[36] Horst, U. (2005), "Stationary Equilibria in Discounted Stochastic Games with Weakly Interacting Players, *Games and Economic Behavior*, 51: 83-108 (2005).

[37] Horst, U. and J. Scheinkman (2002), "Equilibria in Systems of Social Interaction," *Journal of Economic Theory*, forthcoming.

[38] Jack, W. and R. Lagunoff (2004), "Dynamic Enfranchisement," Unpublished version to (2006a): www9.georgetown.edu/faculty/lagunofr/franch10.pdf

[39] Jack, W. and R. Lagunoff (2006a), "Dynamic Enfranchisement," *Journal of Public Economics*, 90: 551-572.

[40] Jack, W. and R. Lagunoff (2006b), "Social Conflict and Gradual Political Succession: An Illustrative Model," *Scandinavian Journal of Economics*, forthcoming.

[41] Judd, K. (2004), "Existence, Uniqueness, and Computatational Theory for Time Consistent Equilibria: A Hyperbolic Discounting Example," mimeo, Hoover Institution.

[42] Krusell, P. and J.-V. Rios -Rull (1999), "On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model," *American Economic Review*, 89: 1156-1181.

[43] Klein, P., P. Krusell, and J.-V. Ríos-Rull (2002), "Time Consistent Public Expenditures," mimeo.

[44] Krusell, P., V. Quadrini, and J.-V. Ríos -Rull (1997), "Politico-Economic Equilibrium and Economic Growth," *Journal of Economic Dynamics and Control*, 21: 243-72.

[45] Krussell, P., V. Quadrini, and J-V Rios-Rull (1996), "Are Consumption Taxes Really Better than Income Taxes," *Journal of Monetary Economics* 37: 475-503.

[46] Krusell, P. and A. Smith (2003), "Consumption-Savings Decisions with Quasi-Geometric Discounting," *Econometrica*, 71: 365-375.

[47] Krusell, P., B. Kuruscu, and A. Smith (2002) "Equilibrium Welfare and Government Policy with Quasi-Geometric Discounting", *Journal of Economic Theory*, 105: 42-72.

[48] Lagunoff (2006b) "Markov Equilibrium in Models of Dynamic Endogenous political rules," mimeo, Georgetown University, `www.georgetown.edu/faculty/lagunofr/dynam-polit-b.pdf`.

[49] Laibson, D. (1997), "Golden Eggs and Hypobolic Discounting," *Quarterly Journal of Economics*, 112, 443-477.

[50] Lizzeri, A., and N. Persico (2004), "Why did the Elites Extend the Suffrage? Democracy and the Scope of Government, with an Application to Britain's 'Age of Reform',' *Quarterly Journal of Economics*, 119(2), 707-765.

[51] Mas-Colell (1985), *The Theory of General Equilibrium: A Differentiable Approach*, Cambridge: Cambridge University Press.

[52] Mertens, J.-F. and T. Parthasarathy (1987), "Equilibria for Discounted Stochastic Games," Core Discussion Paper 8750, Universite Catholique De Lovain.

[53] Messner, M. and M. Polborn (2004), "Voting on Majority Rules, *Review of Economic Studies*, 71: 115-132.

[54] Montrucchio, L. (1987), "Lipschitz Continuous Policy Functions for Strongly Concave Optimization Problems, *Journal of Mathematical Economics*, 16:259-73.

[55] Novak, A.S. (2007), "On Stochastic Games in Economics," *Mathematical Methods of Operations Research*, forthcoming.

[56] Ortega, F. (2005), "Immigration Quotas and Skill Upgrading," *Journal of Public Economics*, 89: 18411863.

[57] Persson, T. and G. Tabellini (2002), *Political Economics*, Cambridge, MA: MIT Press.

[58] Persson, T. and G. Tabellini (2003), *The Economic Effect of Constitutions*, MIT Press: Cambridge.

[59] Persson, T. and G. Tabellini (2007), "Democratic Capital: The Nexus of Political and Economic Change," mimeo.

[60] Rauch, J. (2006) " 'Stoking the Beast': A Comment," *The Atlantic Monthly*, June.

[61] Roberts, K. (1998), "Dynamic Voting in Clubs," mimeo, STICERD/Theoretical Economics Discussion Paper, LSE.

[62] Roberts, K. (1999), "Voting in Organizations and Endogenous Hysteresis," mimeo, Nuffield College, Oxford.

[63] Rothstein, P. (1990), "Order Restricted Preferences and Majority Rule," *Social Choice and Welfare*, 7: 331-42.

[64] Tichenor, D. (2002), *Dividing Lines: The Politics of Immigration Control in America*, Princeton University Press, Princeton, NJ.

[65] Schonhardt-Bailey, C. (2002), "Conservatives Who Sounded Like Trustees But Voted Like Delegates", mimeo, LSE Government Department, October.

[66] Schonhardt-Bailey, C. (2006), *From the Corn Laws to Free Trade. Interests, Ideas, and Institutions in Historical Perspective*, London: The MIT Press.