Games of Status and Discriminatory Contracts *

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Abstract

Following recent empirical evidence which indicates the importance of rank for the determination of workers' wellbeing, this paper introduces status seeking preferences in the form of rank-dependent utility functions into a moral hazard framework with one firm and multiple workers, but no correlation in production. We show that workers' concern for the rank of their wage in the firm's wage distribution induces the firm to offer discriminatory wage contracts when its aim is to induce all workers to expend effort.
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> '...to understand what makes workers satisfied it is necessary to look at the distribution of wages inside a workplace. We show that rank matters to people. They care about where their remuneration lies within the hierarchy of rewards in their office or factory. They want, in itself, to be high up the pay ordering.'
>
> Brown et al. (2005, p. 30)

1 Introduction

There is a wealth of experimental and empirical work that attests to the fact that workers in an organisation care about their position among peers –one recent example is Brown et al. (2005) which provides empirical and experimental evidence on the importance of rank.[1] Yet many of the theoretical results in the literature on optimal incentives in organizations assume completely self interested workers. The question of what happens to the nature of optimal contracts when agents can be *other-regarding* has only recently started getting attention.[2]

In this paper we investigate the case of workers who are status-seeking, in the sense that they care about their *rank*[3] in the reference group. Our focus is on the question of optimal wage contracts in a simple setting of moral hazard with identical status-seeking agents. For simplicity, we follow the literature in making the assumption that status derives only from one dimension, i.e. the order of realized wages although we are aware (as pointed out by Shubik (1971)) that status is often multi-dimensional.

Our main contribution is to show that when risk neutral agents[4] are conscious about their rank, then when workers are not too averse to being behind, the firm finds it worthwhile to use *discriminatory contracts*. This is a surprising result: the firms offers different wage contracts to agents who are ex-ante identical! The intuition behind this result is that firms can exploit the convexity (concavity) in preferences on status so that by offering asymmetric contracts, the gain to the firm in terms of the net savings in expected wages is higher. In particular, with linear preferences on status, asymmetric contracts do no better than symmetric contracts. An implication of our results is that if workers are not identical, i.e. if they have different attitudes

[1]Brown et al. (2005) also provide a comprehensive survey of this literature.

[2]There are some early exceptions e.g. Frank (1984). Itoh (2004) examines moral hazard in incentive contracts when workers have other-regarding preferences, but the analysis is restricted to symmetric contracts only. Neilson and Stowe (2004) studies optimal symmetric linear contracts for workers with other-regarding preferences. Both sets of authors follow Fehr and Schmidt (1999) in modelling other-regarding preferences. The utility functions allow for both *inequality aversion* and *status seeking* (e.g Itoh (2004), Neilson and Stowe (2004)).

[3]Rank is based on the order of realised wages only.

[4]We assume Risk Neutrality in money for simplicity, it is not essential for our results. We only need that utility is additively separable between monetary and status incentives. Workers can have convex or concave preferences on status.

to risk on the status dimension then it would pay firms to let workers self select into different contracts: some with high status but higher probabilities of getting fired, others that are more secure but low status. We also find, consistent with the literature, that when agents are status-seeking, there is *wage compression.*

There are two distinguishing features in our modelling approach. First, we allow the firm to offer asymmetric contracts[5]. A priori if the firm is interested in exploiting incentives from status to reduce its wage cost then not allowing the firm to use asymmetric contracts seems artificially restrictive. Second, differently from the existing literature on other-regarding preferences we do not follow Fehr and Schmidt (1999) in allowing wage levels of other workers and wage differences between workers to matter to workers. We only allow *ordinal* differences to matter. In this we are motivated by two considerations: first there is empirical evidence (Brown et al. (2005)) that employees really care about rank and not about the deviation from a certain reference level, and second, that we would like these preferences to generalise to the case of more than two workers[6]. Indeed, making preferences depend on the whole vector of wages (rather than the order) when there are many co-workers in the reference group requires a lot of information on the part of the worker and seems to require strong assumptions about the specific way in which the wages of co-workers enter the utility function.

Shubik (1971) on the other hand had in mind a much simpler notion of games of status. In the conversion from a two player game to a *game of status* the set of outcomes reduces to essentially three: Win, Lose or Draw. The natural extension of this to many players suggests that what matters is the number of people below, above or at the same rank. Dubey and Geanakoplos (2004) introduce exactly such a utility function for status seeking students. In other words, rank seems to us to be a more robust way to generalize how wage differences matter in the sense that it is an ordinal measure of status and does not require very precise information on the wage distribution[7]. Motivated both by our interest in exploring the role of *rank* as an indicator of status and the simple and general way in which Dubey and Geanakoplos (2004) allow status to matter through ranks, we use a modification of their model of status seeking.

Winter (2004) shows that in an environment of complementarities in production and unobservable efforts, optimal mechanisms may be fully *discriminating*, i.e. they require unequal treatment of equals. The driving force in his story is the coordination

[5]The issue of asymmetric contracts has not been addressed in the literature to the best of our knowledge.

[6]We conjecture that our results generalise for more than 2 workers if there is sufficient richness in the distribution of the stochastic shocks.

[7]It might be argued of course that small changes in wage distribution lead to discontinuous changes in rank, but we could, in principle, add some perceptions of changes in rank so that wages would have to change by a significant amount for rank to change. This point is also addressed in the Conclusion.

problems between multiple agents. The more general point of his model is that when peer effects are important we should observe more "hierarchy" in organizations when this hierarchy is not related to different job descriptions. Our results stem from different peer effects but the flavour is similar.

The paper is organized as follows: Section (2) introduces the model, Section (3) discusses the complete information case, Section (4) contains our main result on asymmetric contracts and finally Section (5) concludes.

2 The Model

In this section we extend the standard model of moral hazard with n agents to allow for status seeking agents. The general model with n workers is presented in A.Dhillon and Herzog-Stein (2006). Here we restrict ourselves to the special case of 2 workers, two effort levels and two states of nature $s_i \in H, L$. Hence $e_i \in \{e^H, e^L\}$ where $e^H > e^L$ and $c_i(e_i) = c(e_i) = \bar{c}$ for $e_i = e^H$ and 0 otherwise.

Let $\mathbf{w} = [w_1, w_2]$. The rank dependent Utility function is:

$$\begin{aligned} u(\mathbf{w}) &= w_i + \beta\hat{\rho}, \text{ if } w_i > w_j \\ u(\mathbf{w}) &= w_i + \beta\tilde{\rho}, \text{ if } w_i = w_j \\ u(\mathbf{w}) &= w_i - \beta\rho', \text{ if } w_i < w_j \end{aligned} \tag{1}$$

where $\beta \geq 0$, denotes the degree to which workers care about rank. We assume $\hat{\rho} \geq \tilde{\rho}, \rho'$, i.e. the utility from being ahead of the other worker is higher than the utility of being at the same level as well as the absolute value of the disutility from being behind the other worker.

Definition: Convex preferences on status are those which satisfy $\hat{\rho} - \tilde{\rho} > \tilde{\rho} + \rho'$, i.e.

$$\hat{\rho} - \rho' > 2\tilde{\rho}.$$

Concave preferences on status are those which satisfy $\hat{\rho} - \tilde{\rho} < \tilde{\rho} + \rho'$, i.e.

$$\hat{\rho} - \rho' < 2\tilde{\rho}.$$

We assume that a worker who rejects the firm's offered contract and therefore does not enter the employment relationship receives a fixed income of zero. Following Neilson and Stowe (2004, p. 10), the natural assumption is made that if a worker is not in an employment relationship then he does not compare his income to that of other workers, and hence the rank dependent component of the utility function is irrelevant. This implies that Worker i's *reservation utility*, $\bar{U}$, is normalized to zero. Finally, we will assume that the utility from rank is bounded:

Assumption 1. The degree to which worker i cares about rank is bounded from above such that

$$\beta < \frac{\bar{c}}{\hat{\rho} + \rho'}.$$

This assumption is made to rule out situations where the worker derives all his utility from status so that he will be ready to work for next to nothing as long as he has status[8]. Since the agent incurs a cost from high effort, we denote the overall utility as $U_i(\mathbf{w}, e_i) = u_i(\mathbf{w}) - \bar{c}$ if he exerts high effort, and $U_i(\mathbf{w}, e_i) = u_i(\mathbf{w})$ otherwise.

Each state of nature is characterized by a different level of output. More specifically the output levels in the two states of nature $s = H, L$ are such that $y^H > y^L$. We do not allow any correlation in agents shocks nor any technological link between them. These assumptions are made to focus on the case when there are no externalities between agents except those induced by status.

The technology describing the stochastic relationship between effort and output is such that if $e_i = e^H$ then with probability $p^{Hs} > 0$ output is $y_i = y^s$ where $s = L, H$. If $e_i = e^L$ instead, then with probability $p^{Ls} > 0$ output is equal to $y_i = y^s$ where $s = L, H$. Thus y_i depends only on the effort level chosen by Worker i and the assumed technology describing the relationship between e_i and y_i, but not on the effort level chosen by Worker j where $i, j = 1, 2$ with $i \neq j$. Define $\Delta p^s \equiv p^{Hs} - p^{Ls}$ for $s = L, H$. Finally, we make the standard assumption that exerting high effort results in a higher probability of the good state:

Assumption 2. The technology is such that the distribution of output if a worker expends effort *first-order stochastically dominates* the distribution of output if a worker expends no effort, i.e.:

$$p^{HL} < p^{LL} \qquad \text{and} \qquad p^{HH} > p^{LH}.$$

The firm possesses $n = 2$ identical production facilities $i = 1, 2$, called factories. Each factory employs exactly one worker. It is assumed that Worker 1 is employed at Factory 1 and Worker 2 at Factory 2. This interpretation of the two workers is made to simplify exposition, so that we can refer to the two factories without confusion. It should not be taken too literally: any two workers in the firm who are not related through production would suffice for our model.

The domain of contracts in our model needs to be specified. In principle we can distinguish between many different feasible sets of contracts. The most general set

[8]This seems to be the situation in British universities which seem to take full advantage of their status seeking academics to pay them a pittance.

is the set of dependent and asymmetric contracts, where wages can be conditioned on the outcome in both factories and on the identity of workers. For the two worker case we have a convenient representation:

$$\begin{array}{ccc} & H & L \\ H & (w_1^{HH}, w_2^{HH}) & (w_1^{HL}, w_2^{HL}) \\ L & (w_1^{LH}, w_2^{LH}) & (w_1^{LL}, w_2^{LL}) \end{array}$$

The matrix denotes the four possible states of nature, a combination of the shocks in each factory. This is the case of dependent contracts: We call a contract *dependent* if wages depend on shocks in both factories. Thus wages are a vector w_i^m where

$$m \in S_D = \{(H,H),(H,L),(L,H),(H,H)\}.$$

Let s_i denote the shock to worker i. A contract is also *dependent and symmetric* if $w_i^{s_i,s_j} = w_j^{s'_j,s'_i}$ when $s^i = s'_j, s^j = s'_i$.

A contract is *independent* if $w_i^{s_i,s_j} = w_i^{s_i}$, i.e. the wages of i are independent of the shock to worker j. Thus wages of worker i are a vector w_i^m where $m \in S_I = \{(H),(L)\}$. Finally a contract is *independent and symmetric* if $w_i^k = w_j^k$.

As is standard in the literature, we assume that the firm is risk-neutral. The firm's choice problem is to choose a profile of wage contracts $\omega^m = w_1^m, w_2^m$ (where m depends on which type of contract is chosen) and a profile of effort levels $\mathbf{e} = e_1, e_2$ which maximize its combined expected profit Π from Factory 1 and 2.

$$\max_{\omega^m,\mathbf{e}} \sum_i \sum_s \left\{p_i^{ls}\left(y_i^s - Ew_i^s\right)\right\} \tag{2}$$

(where Ew_i^s is the expectation over the other factories shocks, given that factory i has shock s.)

The constraints for the firm are:

–the workers' *Individual Rationality* constraints:

$$E\left(U_i(\mathbf{w}, e_i)\right) \geq \bar{U} = 0, \tag{3}$$

for $i = 1,2,\ e_i = e^H, e^L$

–the workers' *Limited Liability* constraints, i.e.

$$w_i^m \geq 0 \tag{4}$$

for $i = 1,2$.

–the *Incentive Compatibility* constraints, i.e, which depend on whether the firm wants high effort or low effort (or a combination) from each worker.

The above profit-maximization problem of the firm is equivalent to the following two-stage problem:

(i) For given effort levels, e_1 and e_2, the firm minimises its expected wage cost EWC.

(ii) The firm maximises expected profit Π by comparing the different outcomes of stage (i) with each other.

In this paper, the focus is on the minimization of the firm's expected wage cost EWC. Stage (ii) is ignored and we assume that the firm wants both workers to exert effort. Suppose contracts are dependent. Then let $Ew_i^{s_i} = \sum_{s_j \in \{H,L\}} \left(p^{Hs_j} w_i^{s_i s_j} \right)$ for $s_i = H, L$. Ew_i^H represents the expected wage of worker i when he has a good shock (expectation is taken over the other workers' shocks) and Ew_i^L represents the expected wage of worker i when he has a bad shock. When contracts are independent, $Ew_i^{s_i} = w_i^{s_i}$. Similarly, let $E\rho(r_i^H) = \sum_{s \in \{H,L\}} p^{Hs} \rho(r_i^{Hs})$ denote the expected utility from rank of worker i when he has shock H, and $E\rho(r_i^L) = \sum_{s \in \{H,L\}} p^{Hs} \rho(r_i^{Ls})$ when he has shock L. Although the rank r_i^s depends on the wage profile, we suppress the argument to save on notation. Then the Incentive Compatibility constraints are:

$$\begin{aligned} & p^{HH}(Ew_i^H + \beta E(\rho(r_i^H)) + (1 - p^{HH})(Ew_i^L + \beta E\rho(r_i^L)) - \bar{c} \\ \geq \; & p^{LH}(Ew_i^H + \beta E(\rho(r_i^H)) + (1 - p^{LH})(Ew_i^L + \beta E\rho(r_i^L)) \end{aligned} \tag{5}$$

With only two states of nature $\Delta p^L = -\Delta p^H$ and hence we can shorten to:

$$\Delta p^H \left[(Ew_i^H - Ew_i^L) + \beta \left(E\rho(r_i^H) - E\rho(r_i^L) \right) \right] - \bar{c} \geq 0 \tag{6}$$

Denote $E\rho(r_i^H) - E\rho(r_i^L)$ as I_i, the *incentives from status.* Then we can rewrite the Incentive Compatibility constraint as the following:

$$\Delta p^H \left[E(w_i^H) - E(w_i^L) \right] \geq \bar{c} - \beta \Delta p^H \left(I_i \right), \tag{7}$$

For the following sections, all proofs are in the Appendix.

3 Complete Information

As a starting point, we assume that effort is observable and verifiable. Even in this situation, we show that firms have incentives to offer asymmetric contracts. We present briefly the standard model and result and then show that with rank dependent preferences the standard results change significantly.

If Worker i's ($i = 1, 2$) preferences are rank-independent ($\beta = 0$) then the whole problem degenerates to the standard complete information problem. We discuss a representative worker i's problem only, since in this case the minimization problem for the two factories is separable and contracts are independent, so that $m \in S_I$. The *complete information optimal contract* $\mathbf{w}^{CI}$ solves the following problem

$$\min_{\mathbf{w}} \sum_{m \in S_I} p^{Hm} w^m \qquad \text{subject to} \qquad \sum_{m \in S_I} p^{Hm} w^m - \bar{c} \geq 0 \text{ and } w^m \geq 0. \tag{8}$$

From (8) it is immediately obvious that there is no unique complete information optimal contract. Instead there is an infinite number of contracts that solve this optimization problem. Any wage contract with $w^m \geq 0$ such that the individual rationality constraint (3) is binding is a complete information optimal contract. The following proposition summarises this standard result of contract theory (see for instance Laffont and Martimort (2002)).

Proposition 1. *(Laffont and Martimort (2002)). Let workers' preferences be rank-independent. Any wage contract $\mathbf{w}^{CI}$ with $w^m \geq 0$ such that*

$$EWC_i\left(\mathbf{w}^{CI}\right) = \bar{c}$$

is a complete information optimal contract. The **first-best cost** *of implementing the high effort level $e^H = 1$ is*

$$C_i^{FB} = EWC_i\left(\mathbf{w}^{CI}\right) = \bar{c}.$$

Thus, any wage contract that leads to a minimum wage cost equal to $\bar{c}$ for each worker and offers a positive wage in each state of nature is an optimal wage contract. All optimal contracts however give the same expected wage cost.

3.1 Rank-Dependence and Complete Information

What happens to this standard result once we introduce rank-dependence? We show that the firm can achieve the minimum feasible costs if it uses symmetric independent contracts when preferences are concave and asymmetric independent contracts when preferences on status are convex. It follows that dependent contracts can do no better.

Observe that the game of status is conceptually a strictly competitive game. Hence giving higher expected status to one worker is at the expense of the other worker who must then be compensated suitably for lower expected status. It is therefore not obvious that costs can be lowered by asymmetric contracts. The problem is not trivial because we have assumed that status only comes from the order of the wages– if there was another dimension of status then, of course, costs can always be lowered, since one worker can be given a higher rank but lower wages and vice versa. However it seems to us highly implausible that high status workers get paid less than low status workers.

The following lemma shows the minimum feasible costs that a firm can achieve.

Lemma 1. *The minimum expected wage cost possible for the firm, EWC^{min} are equal to $2\bar{c} - \beta(\hat{\rho} - \rho')$ if preferences on status are convex and equal to $2(\bar{c} - \beta\tilde{\rho})$ otherwise.*

It can now be established that the firm can achieve minimum expected wage cost EWC^{min} with independent contracts as long as the firm can use asymmetric contracts. Let

$$\bar{p} = \frac{\varphi}{2}\left\{\sqrt{1 + \frac{4}{\varphi}} - 1\right\},$$

where $\varphi = 1 + \frac{\rho'}{\hat{\rho}+\rho'}$. Note, $1 \leq \varphi \leq 3/2$ given the assumptions made about $\hat{\rho}$ and ρ' and hence $0 < \bar{p} < 1$.

Theorem 1. *Assume $0 < p^{HH} < 1$. There exists a profile of optimal independent wage contracts that guarantees minimum expected wage cost EWC^{min} for the firm.*

When preferences on status are concave the contracts are symmetric and equal to

$$w^H = w^L = \bar{c} - \beta\tilde{\rho}.$$

When preferences on status are convex the contracts are asymmetric and equal to $(i, j = 1, 2$ with $i \neq j)$

$$\tilde{\mathbf{w}}_i = \left\{\frac{\bar{c} - \beta\left(p^{HH}\hat{\rho} - (1 - p^{HH})\rho'\right)}{p^{HH}}\ ,\ 0\right\},$$

and $\tilde{\mathbf{w}}_j = \{\tilde{w}, \tilde{w}\}$ where

$$\tilde{w} = \bar{c} - \beta\left(\left(1 - p^{HH}\right)\hat{\rho} - p^{HH}\rho'\right)$$

whenever $p^{HH} \leq \bar{p}$,

and equal to

$$\breve{\mathbf{w}}_i \equiv \left\{ 0,\ \frac{\bar{c} - \beta\left((1 - p^{HH})\hat{\rho} - p^{HH}\rho'\right)}{(1 - p^{HH})} \right\}.$$

and $\breve{\mathbf{w}}_j = \{\breve{w}, \breve{w}\}$ *where*

$$\breve{w} = \bar{c} - \beta\left[p^{HH}\hat{\rho} - (1 - p^{HH})\rho'\right]$$

whenever $p^{HH} \geq 1 - \bar{p}$.

With complete information such that effort is verifiable independent contracts allow the firm to achieve whatever it can achieve with dependent asymmetric contracts as long as it has the ability to choose independent asymmetric contracts. Notice that, unlike the case of rank independent preferences, it is necessary to condition wages on output and not just effort, in order to achieve minimum cost: the asymmetric contract cannot be replicated by a deterministic one that is conditioned only on effort. However, if we allow contracts to be stochastic then we could replicate the asymmetric contract with a symmetric stochastic contract that is conditioned only on effort. In order to provide the same incentives from status, consider a contract that gives worker 1 high wages $w^H > 0$, and worker 2 low wages $w^L = 0$, with probability p^{HH} and vice versa with probability $1 - p^{HH}$ if they both put in high effort. This contract achieves the minimum cost. Thus, in the case of observable effort we find that under some conditions the optimal contract is asymmetric and independent. The intuition behind this result is quite simple − when status and monetary incentives are substitutes, the firm is able to exploit the order of wages to create situations where each worker gets utility from status and hence lowers the total expected wage cost. This intuition is easy to see with the stochastic symmetric contract. Notice that when preferences on status are convex the firm achieves minimum expected cost by making one worker higher in expected status (e.g. in the first case worker i has higher expected status when $p^{HH} > \frac{1}{2}$) but the other worker has higher expected wages. If workers had different types of preferences on status, we could use these results to justify the existence of different contracts: some which have high expected status but a high probability of getting fired and those with low expected status but which are more secure.

This may seem unrealistic: If firms started paying workers according to random events even when effort is fully observed it might cause a loss of morale. However, our results are consistent with situations where effort is unobserved but is very weakly correlated with output. We suggest that in such situations, linking employee compensation differentially to random events (like changes in the value of the firm's stocks) might occur since it lowers overall costs to the firm by creating artificial heirarchies.

Does the presence of status seeking agents cause *wage compression*? Wage Compression in this setting occurs when in the optimal (symmetric independent) contract, the wage difference $|w_i^H - w_i^L|$ is lower with rank dependent preferences than in the benchmark case. In A.Dhillon and Herzog-Stein (2006) we show that given our assumptions on the utility from status ($\hat{\rho} > \rho', \tilde{\rho}$), wage compression holds *independently* of *behindness aversion*[9] as defined in Neilson and Stowe (2004). If preferences on status are concave, then this wage compression result relies on having a positive utility from being equal in rank.

We turn now to the situation when effort is unobservable. What matters here is the expected gain in status when an agent works relative to when he shirks, given that all others are working. The next section addresses the question of whether asymmetric contracts can do better than symmetric contracts, even in the moral hazard setting.

4 Moral Hazard with Risk Neutral parties

If the firm is unable to observe the choice of effort, directly, then the firm can offer only a contract that is contingent on the observable output levels of the two factories.

The remainder of this section is structured as follows. First, the standard moral-hazard problem is presented briefly. As in the complete information case when there is no rank-dependence in utilities, then the two factories problems are separable and so independent symmetric contracts will do as well as the most general contracts.

We then study the moral-hazard problem in an environment of rank-dependent preferences. Here, we present our main result: that asymmetric dependent contracts can achieve the minimum feasible cost. Moreover in A.Dhillon and Herzog-Stein (2006) we show that no independent contract can achieve this cost.

4.1 The Benchmark Moral-Hazard Problem

If Worker i's ($i = 1, 2$) utility is rank-independent, $\beta = 0$, then the whole problem degenerates to the standard moral-hazard problem. For each of its factories, the firm's strategy is to find a wage contract ω_i which minimizes its expected wage cost, and makes the individual worker exert effort at Factory i.[10]

[9] In our model, this is the same as having concave preferences on status.

[10] Therefore in the remainder of this section the subscript i is omitted.

Let $m \in S_I$. With incomplete information, i.e. if Worker i's effort level is not verifiable, the problem of the firm is to find a wage contract $\mathbf{w}$ that minimizes

$$\min_{\mathbf{w}} EWC = \sum_m p^{Hm} w^m,$$

where EWC represents the expected wage cost at Factory i,

–subject to the limited liability constraint (4),

–Worker i's *incentive constraint*:

$$\Delta p^H w^H + \Delta p^L w^L \geq \bar{c}$$

or using the fact that $\Delta p^L = -\Delta p^H$

$$\Delta p^H \left[w^H - w^L \right] \geq \bar{c}. \tag{9}$$

–and worker i's *participation constraint* for low effort:

$$\sum_m p^{Lm} w^m \geq 0$$

Note that this implies the participation constraint for high effort, if the incentive constraints are satisfied.

The following proposition describes the optimal contract for the standard moral-hazard problem. In its main part it repeats Proposition 1 of Itoh (2004).

Proposition 2. *For all $m = L, H$ and risk neutral parties the unique optimal contract solving the standard moral-hazard problem is*

$$\mathbf{w}^S = \left\{ w^H, w^L \right\} = \left\{ \frac{\bar{c}}{\Delta p^H}, 0 \right\},$$

and the expected cost of implementing effort is

$$EWC_i \left(\mathbf{w}^S \right) = \frac{p^{HH} \bar{c}}{\Delta p^H}$$

in Factory i. The firm's overall expected cost of implementing effort is $2EWC_i \left(\mathbf{w}^S \right)$.

4.2 Rank-Dependence and Moral Hazard

If effort is no longer observable and verifiable, and workers possess rank-dependent preferences then the firm's choice problem changes. Like in the standard moral-hazard problem the firm has to base its contracts on an observable variable, output.

However, with rank-dependent preferences, when offering a wage contract to one worker the firm has to take into account the effects of this contract on the other worker. In this section we show our main result: with asymmetric dependent contracts, the firm can achieve the minimum feasible expected wage cost. Moreover no symmetric contract and no independent contract can achieve this cost.

Let $m = (s_1, s_2) \in S_D$ (since this is a dependent contract) and denote $q^m = p^{Hs_1} p^{Hs_2}$, i.e. the joint probability that the shocks are s_1, s_2 in the two factories respectively given that both workers exert high effort.

The firm's problem is to minimize expected wage costs by choosing a dependent wage contract:

$$\min_{\mathbf{w}^m} EWC = \sum_i \sum_m [q^m w_i^m]$$

subject to the limited liability constraints, which follow from (4),

$$w_i^m \geq 0.$$

the incentive constraints (7):

$$\Delta p^H \left[E\left(w_i^H\right) - E\left(w_i^L\right) \right] \geq \bar{c} - \beta E\left(\Delta \rho_i\right), \tag{10}$$

and the (low effort) Individual Rationality constraints ($e_i = 0$)

$$p^{LH} E\left(w_i^H\right) + \left(1 - p^{LH}\right) E\left(w_i^L\right) + \beta E\left(\rho(r_i) | \mathbf{w}\right) \geq 0, \tag{11}$$

where $E\left(\Delta \rho_i\right)$ is Worker i's expected gain or loss in expected rank-utility from expending effort.

Note the Individual Rationality constraints for high effort can be ignored in this framework because they are implied by the (low effort) participation constraints and the incentive constraints.

Recall that I_i denotes the incentives from status. Let $I_1^* + I_2^*$ be the maximized value of $I_1 + I_2$ across different wage orders. The minimum expected wage cost possible for the firm, EWC^{min}, is

$$EWC^{min} = p^{HH} \left\{ 2 \frac{c}{\Delta p^H} - \beta \left(I_1^* + I_2^* \right) \right\}.$$

The minimum feasible cost is achieved when the Expected wage $E(w^L) = 0$ for both workers and the sum of the expected wages $E(w_1^H) + E(w_2^H)$ is just enough to satisfy the sum of incentive constraints. The next lemma characterizes the wage orders that maximize the sum of incentives.

Lemma 2. *The rank payoff matrix which maximizes $I_1 + I_2$ (we can exchange the roles of the two workers and get the same sum of incentives) is given by:*

$$\begin{array}{ccc} & H & L \\ H & (\hat{\rho}, -\rho') & (\hat{\rho}, -\rho') \\ L & (-\rho', \hat{\rho}) & (\tilde{\rho}, \tilde{\rho}) \end{array}$$

when preferences on status are convex, and by

$$\begin{array}{ccc} & H & L \\ H & (\tilde{\rho}, \tilde{\rho}) & (\hat{\rho}, -\rho') \\ L & (-\rho', \hat{\rho}) & (-\rho', \hat{\rho}) \end{array}$$

when preferences on status are concave.

The maximized value of $I_1 + I_2$ is equal to

$$I_1^* + I_2^* = p^{HH}(\hat{\rho} + \rho') + 2(1 - p^{HH})(\hat{\rho} - \tilde{\rho})$$

when preferences on status are convex, and by

$$I_1^* + I_2^* = 2p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho')$$

when preferences are concave.

We can now characterize the optimal asymmetric (dependent) contract. Let

$$\underline{\rho}_1' = \left(\frac{(1 - p^{HH}) \left[p^{LH}\hat{\rho} + (1 - p^{LH})\tilde{\rho} \right]}{p^{HH}} \right)$$

and

$$\underline{\rho}_2' = \left(\frac{p^{LH} \left[(1 - p^{HH})\hat{\rho} + p^{HH}\tilde{\rho} \right]}{1 - p^{LH}} \right)$$

Theorem 2. *Assume that*

$$\rho' \leq \min \left[\underline{\rho}_1', \underline{\rho}_2' \right]. \tag{12}$$

(i) Suppose that preferences are convex: then there exists a dependent contract that achieves the minimum expected cost, EWC^{min}, with an induced asymmetric wage order:

$$w_1^{HH} > w_2^{HH} = 0;\ w_2^{LH} > w_1^{HL} > w_2^{HL} = w_1^{LH} = w_1^{LL} = w_2^{LL} = 0.$$

with

$$E(w_1^H) = \frac{c}{\Delta p^H} - \beta(p^{HH}(\hat{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} - \tilde{\rho}))$$

and

$$E(w_2^H) = \frac{c}{\Delta p^H} - \beta((1 - p^{HH})(\hat{\rho} - \tilde{\rho})).$$

(ii) Suppose preferences are concave: then there exists a dependent contract that achieves an expected wage cost that is $\epsilon > 0$ close to the EWC^{min}:

$$EWC = EWC^{min} + \left(1 - p^{HH}\right)\epsilon,$$

where $\epsilon > 0$[11] with an induced asymmetric wage order:

$$w_1^{HH} = w_2^{HH};\ w_2^{LH} > w_1^{LH} = 0;\ w_1^{HL} > w_2^{HL} = 0;\ w_2^{LL} > w_1^{LL} = 0$$

with

$$E(w_1^H) = \frac{c}{\Delta p^H} - \beta(p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho')),$$

$$E(w_2^L) = \left(1 - p^{HH}\right) w_2^{LL} = \left(1 - p^{HH}\right)\epsilon$$

and

$$E(w_2^H) = \frac{c}{\Delta p^H} - \beta(p^{HH}(\tilde{\rho} + \rho')) + \left(1 - p^{HH}\right)\epsilon,$$

Moreover, we can show that no independent or symmetric contract can achieve these costs[12]. So why is it possible to achieve minimum expected wage cost EWC^{min} with asymmetric dependent contracts but not with symmetric dependent contracts? In the setup here the main trade off is through the incentive constraints, because they are crucial for the determination of the expected wage cost. With asymmetric dependent contracts the incentives from status are higher for worker 1 and lower for worker 2 relative to the situation with the optimal symmetric contract.[13] Denote the asymmetric contract by AS and the symmetric one by S, then

$$I_1^{*AS} > I_1^{*S} = I_2^{*S} > I_2^{*AS} \quad \text{and} \quad E(w)_2^{AS} > E(w)_1^S = E(w)_2^S > E(w)_1^{AS}.$$

However, crucial for the firm's decision to use asymmetric dependent contracts instead of symmetric contracts is whether it can achieve lower expected wage cost with asymmetric dependent than with symmetric contracts. To understand the intuition for the results presented here it is useful to look at the status payoff matrix for a symmetric contract:

[11] We assume that ϵ is the smallest monetary unit to be able to fix the optimal contract; otherwise there is always a smaller ϵ.

[12] Proofs are available in A.Dhillon and Herzog-Stein (2006).

[13] Recall that calling the agent with higher incentives agent 1 is without loss of generality since the roles of the two workers can be exchanged.

	H	L
H	$\tilde{\rho}, \tilde{\rho}$	$\hat{\rho}, -\rho'$
L	$-\rho', \hat{\rho}$	$\tilde{\rho}, \tilde{\rho}$

By offering an asymmetric contract the firm can take advantage of the fact that the changes in incentives (gain for one worker, loss for the other relative to the symmetric contract) are not symmetric for the two workers. Indeed the gain in incentives (relative to the symmetric contract) and hence the reduction in the expected wage cost for one worker must be larger than the loss in incentives (relative to the symmetric contract) and hence the increase in expected wage cost for the other worker. Starting from the optimal symmetric contract there are two states for which a switch to an asymmetric contract can cause a change in incentives if the firm's intention is to do so: the states when either both factories are in the good state or when both factories are in the bad state. First look at the case when both factories are in the good state. When preferences on status are convex then the firm gains more by increasing worker 1' incentives by increasing his status payoff than it looses by reducing worker 2's incentives by reducing his status payoff in the situation when both factories are in the good state, because worker 1's gain in incentives is $I_1^{*AS} - I_1^{*S} = p^{HH} (\hat{\rho} - \tilde{\rho})$ and worker 2's loss in incentives is $I_2^{*AS} - I_2^{*S} = -p^{HH} (\tilde{\rho} + \rho')$. Second, looking at the case when both factories are in the bad state, then when preferences on status are concave the firm gains more by increasing worker 1' incentives by decreasing his status payoff than it looses by reducing worker 2's incentives by increasing his status payoff in the situation when both factories are in the good state, because worker 1's gain in incentives is $I_1^{*AS} - I_1^{*S} = \left(1 - p^{HH}\right) (\tilde{\rho} + \rho')$ and worker 2's loss in incentives is $I_2^{*AS} - I_2^{*S} = -\left(1 - p^{HH}\right) (\hat{\rho} - \tilde{\rho})$. Furthermore with asymmetric dependent contracts in both cases wage profiles exist which translate these overall gains in incentives into lower expected wage cost. If preferences on status were linear, i.e. $\hat{\rho} - \tilde{\rho} = \tilde{\rho} + \rho'$, then asymmetry would not help and the optimal contract would be the symmetric one. Also as shown above with asymmetric independent contracts there are no wage profiles which can translate the overall gains in incentives into the minimum expected wage cost EWC^{min}.

Hence, even when agents are ex-ante symmetric, the firm gains by using discriminatory contracts to exploit the utility that agents get from status. When agents are not symmetric in their status preferences, then the firm could find it optimal to let workers self-select into different contracts. Finally, observe that wage compression occurs in the optimal contract, relative to the standard moral hazard setting, given our assumptions on the utility from status $\hat{\rho} \geq \rho', \tilde{\rho}$.

5 Concluding Remarks

Much of contract theory has focused on agents who are self interested. What happens to the theory of optimal contracts when this assumption is relaxed? In particular, what happens when workers care about their *rank*? We investigate this question in a simple model with two status-seeking agents. We show that then the problem of finding the optimal contract involves: (1) Maximizing the incentives from status given a particular distribution of the stochastic shocks to the two workers: here we are essentially designing a game of status (Shubik (1971)) to maximize the incentives of both workers. (2) Look for the levels of wages that minimize expected wage costs and satisfy the constraints.

Our main results are illustrated in our simple two agent model. If agents are not too averse to being behind, we show that asymmetric contracts or discriminatory contracts where the two agents are offered different contracts even though they are ex-ante identical, are better than symmetric contracts in the case of observable effort, when preferences are convex. In the case of unobservable effort, we find that asymmetric contracts dominate symmetric contracts, as long as the firm is allowed to write down dependent contracts. If the firm is restricted to independent contracts on the other hand, then we show in A.Dhillon and Herzog-Stein (2006) that under some conditions, asymmetric independent contracts are also better than symmetric contracts. Moreover, no symmetric contract *even if it is dependent* can replicate the discriminatory one.

The result relies on the fact that there are enough states of nature that any given agent can be both ahead in some states and behind or equal in others[14]. Second, our result relies on being able to pay one worker just a little bit more than the other (the ϵ in our story). A criticism that might be leveled at us is that people perceive rank differences only when there are noticeable or significant differences in wages (see e.g. Shubik (1971)) . We have two answers to this. First, we refer to anecdotal evidence from Brown et al. (2005) who cite the story of Professor X who refused a job in a top university because he was paid a wage \$ 10 below that of the (then) highest paid professor. In other words, the satisfaction of being top ranked comes from the fact that this hierarchy in wages is common knowledge. This is what we assume in our model (contracts have to be common knowledge to both workers). Second, what we suggest (like Winter (2004)) is that there may be benefits to introducing an artificial hierarchy between workers even when the job is ex-ante identical – again it is the common knowledge about status that is important rather than the actual wage differences. Indeed Baron (1988) suggests that reference actors are people who are nor too different from oneself in terms of pay. Pay differences matter more when

[14]In this respect our result parallels the observation in Dubey and Geanakoplos (2004) that to maximize incentives to students from working some randomness must be introduced in the payoffs from rank. However in our context the randomness is given as part of the moral hazard set up and we can only play with the ranks.

they are across people in the same job title.

Is it better to have status seeking agents as far as the principal is concerned? We might argue that the extra utility that agents get from rank might cause the total expected wages paid out to be *lower* than in the case of agents who are not status seeking (this is the *wage compression* result that is discussed by many authors in this area (e.g. Frank (1984), Neilson and Stowe (2004)). Let us compare symmetric contracts with and without status seeking agents. It turns out that the wage compression result holds in our model both with observable effort and with unobservable effort. This is quite intuitive in that when agents get some utility from rank (in the cases when the state is different across workers), they need to be paid less to exert effort.

We conclude this paper with some ideas for extensions of this work. One obvious extension is to investigate the case of status seeking agents who are not identical (adverse selection). Another interesting question is that of information about wage scales. Why do we observe e.g. that many organizations give broad information about wages (i.e. the bands within which the wages for a given job title lie) but not detailed information (e.g. which employee is getting how much). Is this a case of making the reference group endogenous? How are these bands chosen? We hope to tackle these questions in future work.

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A Appendix: Proof of Lemma 1

Proof: Observe that total costs of the firm are minimized if $E(\rho(r_1)|\mathbf{w})+E(\rho(r_2)|\mathbf{w})$ is maximized. Now,

$$E(\rho(r_1)|\mathbf{w})+E(\rho(r_2)|\mathbf{w})) = (p^{HH})^2(x_1)+(p^{HH})(1-p^{HH})(x_2+x_3)+(1-p^{HH})^2(x_4) \tag{13}$$

where $x_i \in \{\hat{\rho}-\rho', 2\tilde{\rho}\}$ for $i = 1, 2, 3, 4$. It is clear that when preferences are convex $x_i^* = \hat{\rho} - \rho'$ maximizes total expected status while if preferences are convex then $x_i^* = 2\tilde{\rho}$ maximizes total expected status.

□

B Appendix: Proof of Theorem 1

Proof: Suppose preferences on status are concave, then the symmetric independent contract, $w^H = w^L = (\bar{c} - \beta\tilde{\rho})$ implements the EWC^{min}. Note that limited liability and participation constraints are satisfied since $\bar{c} > \beta\hat{\rho}$ follows from Assumption 1.

Now suppose preferences on status are convex. Consider first the profile of asymmetric and independent wage contracts $\tilde{\omega}$. Observe that given the profile $\tilde{\omega}$, $EWC = EWC^{min}$ if the order of wages satisfies:

$$\begin{aligned} \tilde{w}_i^H &> \tilde{w}_j^H \\ \tilde{w}_j^L &> \tilde{w}_i^L, \end{aligned} \tag{14}$$

where, recall that $\tilde{\mathbf{w}}_j = \{\tilde{w}, \tilde{w}\}$ (since then worker i's expected rank utility is $E\left(\rho(r_i)|\tilde{\omega}\right) = p^{HH}\hat{\rho} - (1-p^{HH})\rho'$ and Worker j's expected rank utility is $E\left(\rho(r_j)|\tilde{\omega}\right) = \left(1-p^{HH}\right)\hat{\rho} - p^{HH}\rho'$).

It is sufficient to show that the profile $\tilde{\omega}$ is such that both workers participation constraints are satisfied and order (14) is satisfied:

Worker i's participation constraint is

$$p^{HH}\tilde{w}_i^H + \left(1-p^{HH}\right)\tilde{w}_i^L = \bar{c} - \beta\left[p^{HH}\hat{\rho} - \left(1-p^{HH}\right)\rho'\right] = E\left(\rho(r_i)|\tilde{\omega}\right),$$

and Worker j's participation constraint is

$$\tilde{w} = \bar{c} - \beta\left[\left(1-p^{HH}\right)\hat{\rho} - p^{HH}\rho'\right] = E\left(\rho(r_j)|\tilde{\omega}\right).$$

Hence, the participation constraints are clearly satisfied given wage profile $\tilde{\omega}$.

The order of wages and the limited liability constraints imply that we need to show that $\tilde{w}_i^H > \tilde{w} > \tilde{w}_i^L = 0$.

Given Assumption 1, $p^{HH} \leq \bar{p}$ is a sufficient condition for the first inequality to be fulfilled:

Note, $\tilde{w}_i^H > \tilde{w}$ iff

$$\bar{c} > \beta \left\{ \left[\frac{\left(p^{HH}\right)^2}{1-p^{HH}} \right] \hat{\rho} + \left[1 - \frac{\left(p^{HH}\right)^2}{1-p^{HH}} \right] (-\rho') \right\}.$$

Assumption 1 implies that $\bar{c} > \beta (\hat{\rho} + \rho')$ and hence a sufficient condition for the above (strict) inequality to be fulfilled is

$$(\hat{\rho} + \rho') \geq \left\{ \left[\frac{\left(p^{HH}\right)^2}{1-p^{HH}} \right] \hat{\rho} + \left[1 - \frac{\left(p^{HH}\right)^2}{1-p^{HH}} \right] (-\rho') \right\}.$$

However this (weak) inequality is equivalent to

$$\varphi \geq \left[\frac{\left(p^{HH}\right)^2}{1-p^{HH}} \right] \quad \text{or} \quad p^{HH} \leq \frac{\varphi}{2} \left\{ \sqrt{1 + \frac{4}{\varphi}} - 1 \right\} = \bar{p}.$$

The second inequality follows directly from Assumption 1.

Second. consider the profile of asymmetric and independent wage contracts $\breve{\omega}$. Observe that given the profile $\breve{\omega}$, $EWC = EWC^{min}$ if the order of wages satisfies:

$$\begin{aligned} \breve{w}_i^L &> \breve{w}_j^L \\ \breve{w}_j^H &> \breve{w}_i^H, \end{aligned} \tag{15}$$

where, remember, $\breve{\mathbf{w}}_j = \{\breve{w}, \breve{w}\}$ (since then Worker i's expected rank utility is $E\left(\rho(r_i)|\mathbf{w}\right) = \left(1 - p^{HH}\right)\hat{\rho} - p^{HH}\rho'$. Worker j's expected rank utility is $E\left(\rho(r_j)|\mathbf{w}\right) = p^{HH}\hat{\rho} - \left(1 - p^{HH}\right)\rho'$.)

It is sufficient to show that the profile $\tilde{\omega}$ is such that both workers participation constraints are satisfied and order (15) is satisfied:

Worker i's participation constraint is

$$p^{HH}\breve{w}_i^H + \left(1 - p^{HH}\right)\breve{w}_i^L = \bar{c} - \beta\left[\left(1 - p^{HH}\right)\hat{\rho} - p^{HH}\rho'\right] = E\left(\rho(r_i)|\mathbf{w}\right),$$

and Worker j's participation constraint is

$$\breve{w} = \bar{c} - \beta\left[p^{HH}\hat{\rho} - \left(1 - p^{HH}\right)\rho'\right] = E\left(\rho(r_j)|\mathbf{w}\right).$$

Hence, the participation constraints are clearly satisfied given wage profile $\breve{\omega}$.

The order of wages and the limited liability constraints imply that we need to show that $\breve{w}_i^L > \breve{w} > \breve{w}_i^H = 0$.

Given Assumption 1, $p^{HH} \geq 1 - \bar{p}$ is a sufficient condition for the first inequality to be fulfilled:

Note, $\breve{w}_i^L > \breve{w}$ iff

$$\bar{c} > \beta \left\{ \left[\frac{\left(1 - p^{HH}\right)^2}{p^{HH}} \right] \hat{\rho} + \left[1 - \frac{\left(1 - p^{HH}\right)^2}{p^{HH}} \right] (-\rho') \right\}.$$

Assumption 1 implies that $\bar{c} > \beta \left(\hat{\rho} + \rho'\right)$ and hence a sufficient condition for the above (strict) inequality to be fulfilled is

$$\left(\hat{\rho} + \rho'\right) \geq \left\{ \left[\frac{\left(1 - p^{HH}\right)^2}{p^{HH}} \right] \hat{\rho} + \left[1 - \frac{\left(1 - p^{HH}\right)^2}{p^{HH}} \right] (-\rho') \right\}.$$

However this (weak) inequality is equivalent to

$$\varphi \geq \left[\frac{\left(1 - p^{HH}\right)^2}{p^{HH}} \right] \quad \text{or} \quad p^{HH} \geq 1 - \frac{\varphi}{2} \left\{ \sqrt{1 + \frac{4}{\varphi}} - 1 \right\} = 1 - \bar{p}.$$

The second inequality follows directly from Assumption 1.

□

C Appendix: Proof of Proposition 2

Proof: The Lagrange Function representing the standard moral-hazard problem is

$$L\left(\mathbf{w}, \lambda\right) = p^{HH} w^H + \left(1 - p^{HH}\right) w^L + \lambda \left[\bar{c} - \Delta p^H \left[w^H - w^L\right]\right]. \tag{16}$$

Minimising (16) leads to the following Kuhn-Tucker conditions:

$$p^{HH} - \lambda \Delta p^H \geq 0 \quad \text{and} \quad w^H \geq 0 \quad \text{and} \quad w^H \left[p^{HH} - \lambda \Delta p^H\right] = 0$$

$$\left(1 - p^{HH}\right) + \lambda \Delta p^H \geq 0 \quad \text{and} \quad w^L \geq 0 \quad \text{and} \quad w^L \left[\left(1 - p^{HH}\right) + \lambda \Delta p^H\right] = 0$$

and

$$\left[\bar{c} - \Delta p^H \left[w^H - w^L\right]\right] \leq 0 \quad \text{and} \quad \lambda \geq 0 \quad \text{and} \quad \lambda \left[\bar{c} - \Delta p^H \left[w^H - w^L\right]\right] = 0.$$

These Kuhn-Tucker conditions imply that the unique solution is equal to $w^L = 0$, $w^H = \frac{\bar{c}}{\Delta p^H}$, and $\lambda = \frac{p^{HH}}{\Delta p^H}$. The results of Proposition 2 follow immediately.

□

D Appendix: Proof Of Lemma 2

Proof:

Step 1: Consider the matrix:

	H	L
H	(x_1, y_1)	(x_2, y_2)
L	(x_3, y_3)	(x_4, y_4)

The problem is to maximize the following expression, by choice of (x_i, y_i) pairs:

$$I_1 + I_2 = p^{HH}\left((x_1 - x_3) + (y_1 - y_2)\right) + \left(1 - p^{HH}\right)\left((x_2 - x_4) + (y_3 - y_4)\right).$$

subject to the constraint that $(x_i, y_i) \in \{(\hat{\rho}, -\rho'); (\tilde{\rho}, \tilde{\rho}); (-\rho', \hat{\rho})\}$. Notice that we must have $(x_2, y_2) = (\hat{\rho}, -\rho')$ and $(x_3, y_3) = (-\rho', \hat{\rho})$ since $I_1 + I_2$ is increasing in x_2, y_3 and decreasing in x_3, y_2.

Hence the rank payoff matrix which maximizes $I_1 + I_2$ is of the form:

	H	L
H	(x_1, y_1)	$(\hat{\rho}, -\rho')$
L	$(-\rho', \hat{\rho})$	(x_4, y_4)

Step 2: The various possibilities given Step 1 above are the following (we can exchange the roles of the two players and get the same sum of incentives):

(A) $(x_1, y_1) = (\hat{\rho}, -\rho')$, $(x_4, y_4) = (-\rho', \hat{\rho})$, with $I_1 + I_2 = \hat{\rho} + \rho'$

(B) $(x_1, y_1) = (\hat{\rho}, -\rho')$, $(x_4, y_4) = (\tilde{\rho}, \tilde{\rho})$ with $I_1 + I_2 = p^{HH}(\hat{\rho} + \rho') + 2(1 - p^{HH})(\hat{\rho} - \tilde{\rho})$.

(C) $(x_1, y_1) = (\tilde{\rho}, \tilde{\rho})$, $(x_4, y_4) = (-\rho', \hat{\rho})$, with $I_1 + I_2 = 2p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho')$.

(D) $(x_1, y_1) = (\tilde{\rho}, \tilde{\rho})$ $(x_4, y_4) = (\tilde{\rho}, \tilde{\rho})$ with $I_1 + I_2 = 2p^{HH}(\tilde{\rho} + \rho') + 2(1 - p^{HH})(\hat{\rho} - \tilde{\rho})$.

It is easy to see that the matrix which maximizes $I_1 + I_2$ is (B) when preferences are convex and (C) when preferences are concave.

□

E Appendix: Proof of Theorem 2

Proof: Suppose that preferences are convex. It is clear that (i) implements the rank payoff matrix (B), since

$$w_1^{HH} > w_2^{HH} = 0; 0 = w_2^{HL} < w_1^{HL}; 0 = w_1^{LH} < w_2^{LH}; w_1^{LL} = w_2^{LL} = 0.$$

This implies $0 = E(w_1^L) = E(w_2^L)$. Also, with the wages given, we need to show that the incentive compatibility constraints are satisfied: Indeed, both incentive constraints are binding given that $E(w_i^L) = 0$ and $E(w_i^H) = \frac{c}{\Delta p^H} - \beta I_i$ for $i = 1, 2$. Since $I_1 > I_2$ this implies that $E(w_1^H) < E(w_2^H)$. This is satisfied by choosing w_2^{LH} to satisfy $\left(1 - p^{HH}\right) w_2^{LH} = E(w_2^H)$. The limited liability constraints for worker 1 are satisfied if $\frac{c}{\Delta p^H} \geq \beta(p^{HH}(\hat{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} - \tilde{\rho}))$, but this is the case since we assumed that $\frac{c}{\Delta p^H} > \beta\left(\hat{\rho} + \rho'\right)$. Since $E(w_2^H) > E(w_1^H) > 0$ the limited liability constraints for worker 2 are satisfied, too. It remains to check that the participation constraints are satisfied.

We need the following condition for the participation constraint (low effort) for worker 1 to hold:

$$\begin{aligned} p^{LH} E(w_1^H) + \left(1 - p^{LH}\right) E(w_1^L) + \beta \Big[p^{LH}\left(\hat{\rho}\right) &+ \\ \left(1 - p^{LH}\right)\left(p^{HH}(-\rho') + (1 - p^{HH})(\tilde{\rho})\right) \Big] &\geq 0, \end{aligned} \tag{17}$$

Since $p^{LH} E(w_1^H) + \left(1 - p^{LH}\right) E(w_1^L) > 0$ it is sufficient to check that

$$p^{LH}\left(\hat{\rho}\right) + \left(1 - p^{LH}\right)\left(p^{HH}(-\rho') + (1 - p^{HH})(\tilde{\rho})\right) \geq 0.$$

Similarly for worker 2 we need:

$$p^{LH} E(w_2^H) + \left(1 - p^{LH}\right) E(w_2^L) + \beta\left[p^{HH}\left(-\rho'\right) + \left(1 - p^{HH}\right)\left(p^{LH}(\hat{\rho}) + (1 - p^{LH})(\tilde{\rho})\right)\right] \geq 0, \tag{18}$$

Both of these are satisfied given our assumptions on ρ' being sufficiently small. Note, that if ρ' is sufficiently small such that

$$p^{HH}\left(-\rho'\right) + \left(1 - p^{HH}\right)\left(p^{LH}(\hat{\rho}) + (1 - p^{LH})(\tilde{\rho})\right) \geq 0,$$

then worker 2's and worker 1's participation constraint are both fulfilled. In particular, if $\rho' = 0$ the participation constraints are always satisfied.

(ii)

Suppose that preferences are concave. It is clear that (ii) implements the rank payoff matrix (C), since

$$w = w_1^{HH} = w_2^{HH}; 0 = w_2^{HL} < w_1^{HL}; 0 = w_1^{LH} < w_2^{LH}; 0 = w_1^{LL} < w_2^{LL} = \epsilon.$$

This implies that $0 = E(w_1^L) < E(w_2^L) = (1 - p^{HH})\epsilon$. Also, with the wages given, we need to show that the incentive compatibility constraints are satisfied: Note that $E(w_1^L) = 0$, hence worker 1's incentive constraint is binding. Since $E(w_2^L) = \epsilon$, worker 2's incentive constraint is also binding. Since $I_1 > I_2$ this implies that $E(w_1^H) < E(w_2^H)$. This is satisfied by the choice of w_2^{LH}, such that $E(w_2^H) = \left(p^{HH} w + (1 - p^{HH}) w_2^{LH}\right)$.

The limited liability constraints are satisfied if $\frac{c}{\Delta p^H} \geq \beta(p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho'))$, but this is the case since we assumed that $\frac{c}{\Delta p^H} > \beta(\hat{\rho} + \rho')$. Since $E(w_2^H) > E(w_1^H) > 0$ the limited liability constraints for worker 2 are satisfied. It remains to check that the participation constraints are satisfied.

We need the following condition for the participation constraint for worker 1 to hold:

$$p^{LH} E(w_1^H) + \left(1 - p^{LH}\right) E(w_1^L) + \beta \Big[p^{LH} \left(p^{HH} \tilde{\rho} + (1 - p^{HH})\hat{\rho}\right) + \left(1 - p^{LH}\right)(-\rho')\Big] \geq 0, \tag{19}$$

Since $p^{LH} E(w_1^H) + \left(1 - p^{LH}\right) E(w_1^L) > 0$ it is sufficient to check that

$$p^{LH} \left(p^{HH} \tilde{\rho} + \left(1 - p^{HH}\right) \hat{\rho}\right) + \left(1 - p^{LH}\right)(-\rho') \geq 0,$$

Similarly for worker 2 we need:

$$\begin{aligned} p^{LH} E(w_2^H) + \left(1 - p^{LH}\right) E(w_2^L) + \beta \Big[p^{LH} \left(p^{HH} \tilde{\rho} + \left(1 - p^{HH}\right) \hat{\rho}\right) \quad + \\ \left(1 - p^{LH}\right) \left(p^{HH}(-\rho') + (1 - p^{HH})(\hat{\rho})\right)\Big] \quad \geq \quad 0, \end{aligned} \tag{20}$$

Both of these are satisfied given our assumptions on ρ' being sufficiently small. Note, that if ρ' is sufficiently small such that

$$p^{LH} \left(p^{HH} \tilde{\rho} + \left(1 - p^{HH}\right) \hat{\rho}\right) + \left(1 - p^{LH}\right)(-\rho') \geq 0,$$

then worker 1's and worker 2's participation constraint are both fulfilled. In particular, if $\rho' = 0$ the participation constraints are always satisfied.

□