Lecture 2: Learning and Fiscal Policy

1 Introduction

• In contrast to research in learning and monetary policy, the literature on learning and fiscal policy is fairly small. Recently much more interest.
2 Fiscal Policy in RBC Models: Learning View

• What are the economic effects of cuts in capital taxation? Giannitsarou (2006) compares the dynamic effects under RE and under learning.

• The model is quite standard: households maximize

\[ E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t + AL_t), \]  subject to

\[ C_t + K_t - K_{t-1} = (1 - \tau)(R_t - \delta)K_{t-1} + (1 - \xi)W_tN_t, \]

where \( N_t = 1 - L_t \) and \( \tau \) and \( \xi \) are tax rates on capital and labor, respectively. Evolution of capital is

\[ K_t = (1 - \delta)K_{t-1} + I_t. \]
The firm maximizes profits

\[ Y_t - W_t N_t - R_t K_{t-1}, \text{ subject to } Y_t = Z_t K_{t-1}^\alpha N_t^{1-\alpha}, \]

where \( 0 < \alpha < 1 \). The technology shock follows

\[ \log Z_t = \rho \log Z_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_\varepsilon^2), 0 < \rho < 1. \]

Government expenditures \( G_t \) are balanced by taxes

\[ G_t = \tau (R_t - \delta) K_{t-1} + \xi W_t N_t. \]

- The REE is summarized by equations

\[ 1 = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left[ (1 - \tau)(R_{t+1} - \delta) + 1 \right] \right] \]

\[ AC_t = (1 - \xi)W_t, K_{t-1}R_t = \alpha Y_t, W_t N_t = (1 - \alpha)Y_t \]
\[ C_t + K_t = Y_t + (1 - \delta)K_{t-1} - G_t \]

\[ G_t = \tau(R_t - \delta)K_{t-1} + \xi W_t N_t. \]

\[ Y_t = Z_t K^\alpha_{t-1} N_t^{1-\alpha}, \quad \log Z_t = \rho \log Z_{t-1} + \varepsilon_t. \]

2.1 Linearized Model

- Let \( \bar{X} \) be the nonstochastic steady state of a variable and \( x_t = \log X_t - \log \bar{X} \). The log-linearized equations are

\[ 0 = E_t[c_t - c_{t+1} + (1 - \beta + \beta \delta (1 - \tau) r_t)], \quad 0 = w_t - c_t \]

\[ c_t = \frac{\bar{Y}}{C} y_t - \frac{\bar{G}}{C} g_t + \frac{1 - \delta}{C} \bar{K} k_{t-1} - \bar{K} k_t \]
\[ r_t = z_t - (1 - \alpha)k_{t-1} + (1 - \alpha)n_t \]

\[ y_t = z_t + \alpha k_{t-1} + (1 - \alpha)n_t, \quad w_t = z_t + \alpha k_{t-1} - \alpha n_t \]

\[ g_t = \frac{\tau \bar{R}K}{G}(r_t + k_{t-1}) - \frac{\tau \delta K}{G}k_{t-1} + \frac{\xi \bar{W} \bar{L}}{G}(w_t + n_t). \]

- The reduced form is

\[
\begin{align*}
  k_t &= a_1 E_t k_{t+1} + a_2 k_{t-1} + b z_t \\
  z_t &= \rho z_{t-1} + \varepsilon_t.
\end{align*}
\]

Letting \( x_t = (k_t, z_t)' \), the MSV REE is of the form

\[
 k_t = \Phi' x_{t-1} + \theta \varepsilon_t = \phi_1 k_{t-1} + \phi_2 z_{t-1} + \theta \varepsilon_t
\]
and by method of undetermined coefficients

\[
\Phi'_{+,-} = \left( \frac{1}{2a_1} (1 \pm \sqrt{1 - 4a_1a_2}), \frac{\rho b}{1 - a_1(\rho + \phi_1)} \right)'.
\]

Now \( \Phi_- \) is the unique stationary solution when \( |a_1 + a_2| < 1 \). If \( |a_1 + a_2| > 1 \) and \( |a_1a_2| < 1/4 \), both solutions are stationary. (In other cases both solutions are nonstationary or no-real.)

### 2.2 Learning

- See EH-book, chapter 10 for basics of learning in RBC models. Agents have PLM of the MSV functional form. The forecast is

\[
E_t^* k_{t+1} = \phi_1 k_t + \phi_2 z_t
\]
and the ALM is

\[ k_t = T(\Phi_{t-1})'x_{t-1} + V(\Phi_{t-1})\varepsilon_t, \]

where

\[
T(\Phi) = \left((1 - a_1\phi_1)^{-1}a_2, (1 - a_1\phi_1)^{-1}(\rho a_1\phi_2 + b\rho)\right)'
\]
\[
V(\Phi) = (1 - a_1\phi_1)^{-1}(a_1\phi_2 + b) .
\]

The updating is done by usual RLS. Learning converges locally to \( \Phi^* \) if

\[
\frac{a_1a_2}{(1 - a_1\phi_1)^2} < 1 \quad \text{and} \quad \frac{\rho a_1}{1 - a_1\phi_1} < 1 .
\]

If \(|a_1 + a_2| < 1\) then convergence is to \( \Phi^- \).
2.3 Policy Change

- Unanticipated tax change at time $t = 0$ from $\tau^{old}$ to $\tau^{new}$. Assume that the economy was in the REE before the change. The calibration: $A = 2.54$, $\alpha = 0.36$, $\delta = 0.025$, $\beta = 0.99$, $\rho = 0.95$ and $\sigma_\varepsilon = 0.00712$. $\tau$ and $\xi$ are varied in $[0, 1)$. With this calibration $\Phi_-$ is unique stationary REE.

- To simulate REE dynamics:
  (a) calculate old and new steady states,
  (b) log-linearize around new steady state and find $\Phi^{new}$
  (c) set initial condition for capital and simulate the $k_t$ path
  (d) compute the paths of the rest of the variables.
For taxes set $\tau^{old} = 0.4$ and $\xi = 0.25$. The tax change is $\tau^{new} = 0$. The steady state change is

<table>
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<tr>
<th></th>
<th>$\bar{Y}$</th>
<th>$\bar{K}$</th>
<th>$\bar{C}$</th>
<th>$\bar{I}$</th>
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<td>large:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$old$</td>
<td>1.1186</td>
<td>9.6261</td>
<td>0.6341</td>
<td>0.2406</td>
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<tr>
<td>$new$</td>
<td>1.1994</td>
<td>12.3014</td>
<td>0.6999</td>
<td>0.3075</td>
</tr>
</tbody>
</table>

Under REE the usual impulse response analysis of a shock.

To simulate learning dynamics:

(e) assume a gain sequence, $\gamma_t = 1/(t + 40)$ was used,
(f) fix initial perceptions $\Phi_{-1} = \bar{\Phi}^{old}$ and also
$S_{-1} = \bar{M}^{old} = \lim_{t \to \infty} E [x_t(\bar{\Phi}^{old})x_t(\bar{\Phi}^{old})']$. 
• (i) Now agents try to learn the coefficient of the dynamics. Parameter evolution must be incorporated. Different cases of shocks and initial conditions.

(ii) Giannitsarou finds that tax reform when there is a negative technology shock leads to very different paths under learning and under RE dynamics.

(iii) The negative technology shocks can sometimes send the PLM parameters initially adjusting in the wrong direction. Adjustment of capital stock slowed down, consumption and output can even drop initially.

(iv) In the long run the learning economy also finds the new REE.
Fig. 1. The effect of the shock to technology. Solid lines show evolution under adaptive learning and dashed lines show evolution under RE.
Fig. 6. Simulation of output with recessionary shocks.
2.4 Other issues of fiscal policy and learning

- Balanced Budget Rules in Fiscal Policy

- Interactions between fiscal and monetary policy:
  - fiscal theory of prices and “fiscalist equilibria”: see Evans and Honkapohja (2007), Eusepi and Preston (2010),
  - literature on zero interest rate lower bound (Evans and Honkapohja RED 2005, Evans, Guse and Honkapohja EER 2008).
3 Ricardian Equivalence Under Learning

Ricardian Equivalence (RicE) Proposition is one of the most prominent theories in modern macroeconomics.

- Well-known limitations to the RicE result:
  - agents are not dynamic optimizers, households are liquidity constrained, taxes are distortionary, government spending is endogenous, etc.

- The significance of the rational expectation (RE) assumption for RicE has not been examined.
  - Some suggestions that uncertainties and misperceptions about future taxes and government spending lead to failure of the RicE proposition (Feldstein 1982, Seater 1993).
Questions:
  $$\Rightarrow$$ Will RicE still hold if expectations are not fully rational?

Our answer: in usual standard settings RicE holds also outside RE equilibrium.

Some new reasons for failure of RicE:
  - The out-of-equilibrium government budget is partly balanced by expenditure changes.
  - The information set used by agents in forecasting may include government financing variables.
3.1 The Model

- We use the standard discrete-time Ramsey model with a large number of identical households.

- *Assumption 1*: Households choose $c_t$ to maximize their intertemporal expected utility, for given expectations about interest rates, real wages, and taxes.

$$\max \ E_t^* \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{c_s^{1-\sigma}}{1-\sigma} \right\} \ \text{subject to} \ a_{s+1} = w_s + r_s a_s - c_s - \tau_s, \ \ (1)$$

for given $a_t$.

$\sigma > 0$ and $0 < \beta < 1$. $c_s$, $\tau_s$, $w_s$ are consumption, taxes and the real wage rate in period $s$, and $r_s$ is the gross real rate of return on both capital and government one-period bonds at the beginning of period $s$. 
• Per capita household wealth \( a_s \equiv k_s + b_s \). Labor supply is equal to 1, and a No Ponzi Game condition imposed.

• The Euler equation for consumption is

\[
   c_t^{-\sigma} = \beta E_t^* (r_{t+1} c_{t+1}^{-\sigma}).
\]

Notation: \( c_{t+1}^e(t) = E_t^* c_{t+1} \) for expectations. Forward substitution gives

\[
   c_{t+j}^e(t) = c_t \beta^{\frac{j}{\sigma}} (D_{t,t+j}^e(t))^{\frac{1}{\sigma}}, \text{ where } D_{t,t+j}^e(t) = \prod_{i=1}^{j} r_{t+i}^e(t), j \geq 1.
\]

• \( \tau_{t+j}^e(t) \) and \( a_{t+j}^e(t) \) are the expected lump-sum tax and net assets in period \( t + j \). The transversality condition holds

\[
   \lim_{T \to \infty} (D_{t,t+T-1}^e(t))^{-1} a_{t+T}^e(t) = 0.
\]
• The intertemporal budget constraint of the consumer,

\[ 0 = r_t a_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e(t))^{-1}(w_{t+j}^e(t) - c_{t+j}^e(t) - \tau_{t+j}^e(t)) + w_t - c_t - \tau_t, \]

for given expectations \( \{r_{t+j}^e(t)\}, \{w_{t+j}^e(t)\}, \{\tau_{t+j}^e(t)\} \).

The consumption function is

\[ c_t(1 + S_D^e(t)) = r_t a_t + w_t - \tau_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e(t))^{-1}(w_{t+j}^e(t) - \tau_{t+j}^e(t)), \]

where

\[ S_D^e(t) \equiv \sum_{j=1}^{\infty} \beta^{j/\sigma} (D_{t,t+j}^e(t))^{\sigma^{-1}-1}. \]
• Rewrite the consumption function as

\[ c_t(1 + S_D^e(t)) = r_t a_t + PV_t^e(w) - PV_t^e(\tau), \]  

(6)

where

\[ PV_t^e(w) = w_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e(t))^{-1} w_{t+j}^e(t) \]  

and

\[ PV_t^e(\tau) = \tau_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e(t))^{-1} \tau_{t+j}^e(t) \]

are assumed to be finite.

• The production function in intensive form is \( y_t = f(k_t) \), where \( Y_t \) is output, \( K_t \) is capital, \( L_t \) is labour, \( k_t = K_t / L_t \), \( y_t = Y_t / L_t \), and \( f(k) \equiv F(k, 1) \); \( F(K, L) \) is CRS.
• The Cobb-Douglas form \( f(k) = k^\alpha \) is assumed in the examples. For simplicity, no productivity growth and \( 0 \leq \delta \leq 1 \) denotes depreciation. Profit maximization by firms lead to

\[
\begin{align*}
    w_t &= f(k_t) - k_t f'(k_t), \quad (7) \\
    r_t &= 1 - \delta + f'(k_t). \quad (8)
\end{align*}
\]

• The government’s flow budget constraint is

\[
b_{t+1} + \tau_t = g_t + r_t b_t, \quad (9)
\]

where \( g_t \) is government purchases of the good. Market clearing determines \( k_{t+1} \) from

\[
k_{t+1} = f(k_t) - c_t - g_t + (1 - \delta) k_t. \quad (10)
\]
• Given pre-determined variables $k_t, b_t$, current fiscal policy variables $g_t, \tau_t$, and expectations $\{r_{t+j}^e(t)\}_{j=1}^{\infty}, \{w_{t+j}^e(t)\}_{j=1}^{\infty}, \{\tau_{t+j}^e(t)\}_{j=1}^{\infty}$, a temporary equilibrium at time $t$ is determined by the consumption function, equations for the wage rate and the interest rate, the government budget constraint, and market clearing.

Now introduce the present value of government spending

$$PV_t^e(g) = g_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e(t))^{-1} g_{t+j}(t)$$

and we assume:

**Assumption 2:** Households’ forecasts (i) are consistent with the flow budget constraint (9) and (ii) satisfy

$$\lim_{j \to \infty} (D_{t,t+j}^e(t))^{-1} b_{t+j+1}^e(t) = 0.$$
Lemma 1. Under Assumptions 1 and 2, the consumption function (6) can be written as

\[ c_t(1 + S^e_D(t)) = r_t k_t + PV^e_t(w) - PV^e_t(g). \]  \hspace{1cm} (11)

- Consequences:
  (i) The temporary equilibrium can equivalently be defined with government spending \( \{g^e_{t+j}(t)\}_{j=1}^{\infty} \), in place of taxes \( \{\tau^e_{t+j}(t)\}_{j=1}^{\infty} \).
  (ii) With some additional assumptions, RicE holds in the temporary equilibrium.
3.2 Ricardian Equivalence for Paths of Temporary Equilibria with Learning

- Start with some initial capital $k_0$ and public debt $b_0$. With learning, the economy evolves along a path of temporary equilibria $\{c_t, k_{t+1}, r_{t+1}, w_{t+1}, b_{t+1}\}_{t=0}^\infty$ for given fiscal policy rules that determine $\{(g_t, \tau_t)\}_{t=0}^\infty$.

- Under Assumptions 1 and 2, the consumption function (11) depends on $\{r_{t+j}^e(t), w_{t+j}^e(t), g_{t+j}^e(t)\}_{j=1}^\infty$. We assume:

*Assumption 3:* Government spending $g_t$ either is exogenous or is a predetermined variable that depends only on $\{k_s, r_s, w_s, g_{s-1}\}_{s=0}^t$. 
Next, introduce the concept of a learning mechanism and an assumption about the associated information set:

**Definition:** An adaptive learning mechanism is a mapping from the time \( t \) information set \( I_t \) to the sequence \( \{r_{t+j}^e(t), w_{t+j}^e(t), g_{t+j}^e(t)\}_{j=1}^\infty \), for \( t \geq 0 \), together with an initial set of expectations

\[
\{r_j^e(-1), w_j^e(-1), g_j^e(-1)\}_{j=0}^\infty.
\]

**Assumption 4:** The information set at \( t \geq 0 \) for the adaptive learning mechanism consists of

\[
I_t = \{k_s, r_s, w_s, g_s\}_{s=0}^t \cup \{r_{s+j}^e(s), w_{s+j}^e(s), g_{s+j}^e(s)\}_{j=1}^\infty\}_{s=-1}^{t-1}.
\]
Definition: Given $k_0$ (and hence $r_0, w_0$), $b_0$, $g_{-1}$ and initial expectations \( \{r_j^e(-1), w_j^e(-1), g_j^e(-1)\}_{j=0}^{\infty} \), a path of (temporary) equilibria with learning is a sequence \( \{c_t, g_{t+1}, k_{t+1}, r_{t+1}, w_{t+1}, b_{t+1}\}_{t=0}^{\infty} \) of temporary equilibria, in which expectations \( \{r_{t+j}^e(t), w_{t+j}^e(t), g_{t+j}^e(t)\}_{j=1}^{\infty} \) at each time $t$ that satisfy an adaptive learning mechanism.

Definition: The model exhibits Ricardian Equivalence (RicE) if, for all initial conditions, the sequence of variables \( \{c_t, k_{t+1}, r_{t+1}, w_{t+1}\}_{t=0}^{\infty} \) along the path of equilibria with learning is independent of the government financing policy.

Theorem 2 The Ramsey model exhibits Ricardian Equivalence under Assumptions 1 through 4.
3.3 Examples

Example 1: (exogenous government spending). Government spending is constant. Initial public debt is zero. Two cases: (i) balanced budget financing. (ii) taxes are held below \( \bar{g} \) for \( T \) periods, after which debt is stabilized.

- For the learning rules it is assumed that agents have “steady-state” adaptive learning rules with constant gain:

\[
\begin{align*}
    r_{t+i}^e(t) &= r^e(t) \text{ where } r^e(t) = r^e(t-1) + \gamma(r_t - r^e(t-1)) \text{ and } \\
    w_{t+i}^e(t) &= w^e(t) \text{ where } w^e(t) = w^e(t-1) + \gamma(w_t - w^e(t-1)) ,
\end{align*}
\]

for all \( i = 1, 2, \ldots \) and for all \( t = 0, 1, 2, \ldots \).
- **Remark**: More generally, agents’ forecasts of future values of $r_t$ and $w_t$ might depend on the capital stock. In stochastic settings this dependence can be estimated using least-squares adaptive learning rules. However, in the nonstochastic setting it is not possible for the agents to estimate both intercept and slope parameters.

- If initial conditions $k_0$ and $b_0$ and, in the case of learning, initial expectations $r^e(-1)$ and $w^e(-1)$, are not at the steady state, the path of temporary equilibria with learning will differ from the RE path. Nevertheless, RicE holds under learning (and also under RE).
Example 2: (endogenous government spending). We now assume

\[ g_t = \begin{cases} 
\bar{g} & \text{for } 0 \leq t < T \\
\bar{g} + (\beta^{-1} - r_t)b_T & \text{for } t \geq T 
\end{cases} \]

\[ \tau_t = \begin{cases} 
\bar{\tau} & \text{for } 0 \leq t < T \\
\bar{g} + (\beta^{-1} - 1)b_T & \text{for } t \geq T 
\end{cases} \]

For simplicity, the economy is initially in the steady state, i.e. \( k_0 = \bar{k} \) and \( b_0 = 0 \).

- In Example 2, after period \( T \) taxes pay for the permanent level of government spending and the steady-state interest on public debt. Under learning any deviation between the steady-state and actual interest rates on debt is paid for by a corresponding adjustment in government spending.
Numerical illustration: initial expectations at $w^e(-1) = w^e(0) = \bar{w}$, while $r^e(0) = 1.01r_0$. Parameter values: $\sigma = 1$, $\beta = 0.95$, $\delta = 0.1$, $\alpha = 1/3$, $\bar{g} = 0.4$, $\bar{\tau} = 0.3$, $T = 20$, and $\gamma = 1/40$ in the learning rule. As $\bar{\tau} < \bar{g}$, we have debt financing (thick solid curves).

Figure: Paths of consumption (left-hand figure) and capital (right-hand figure) for learning under deficit financing (solid line) and under balanced budget (thin line) when government spending is endogenous. The horizontal dashed line shows the RE steady-state path.
• Under learning RicE fails for this fiscal regime even though RicE holds under RE. The reason for failure stems from the endogeneity of government spending with respect to the interest rate and the positive level of debt.

Example 3: (expectations depend on debt)

Interest rate expectations depend on changes in the level of public debt:

\[ r^e(t) = r^e(t - 1) + \gamma(r_t - r^e(t - 1)) + \eta(b(t) - b(t - 1)). \]  

(12)

Everything else is the same as in Example 1. The case of \( \eta > 0 \) can be viewed as reflecting the belief that rising levels of public debt may lead to higher future real interest rates.
The parameter values are $\sigma = 1$, $\beta = 0.99$, $\delta = 0.03$, $\alpha = 1/3$, $\bar{g} = 0.4$, $\bar{\tau} = 0.3$, $T = 20$, and $\gamma = 1/20$. Initial conditions are set at $b_0 = 0$, $k_0 = \bar{k}$, $r^e(-1) = \bar{r}$ and $w^e(-1) = \bar{w}$ and let $\eta = 0.002$.

Figure 4: Paths of consumption (left-hand figure) and capital (right-hand figure) under learning for deficit financing (solid curve) and balanced budget (horizontal thick line) when the learning rule depends on debt.
4 Anticipated Fiscal Policy and Learning

- One of RE contributions is the idea that agents can anticipate the effects of an announced policy shift in the future. (Sargent and Wallace 1973 etc.)

- So far, the literature on learning has considered effects of unanticipated policy shifts (e.g. Evans, Honkapohja & Marimon 2001, Marcet & Nicolini 2003, Giannitsarou 2006).

- Dynamics from anticipated policy shifts when agents have limited structural knowledge.
  - Particular case: agents know the government budget constraint and the announced policy shift, but they must forecast future prices.
4.1 The Model

- Identical households, who do not know that they are identical. Household problem:

\[
Max \ E_t^* \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{c_s^{1-\sigma}}{1-\sigma} \right\} \text{ s.t.}
\]

\[
b_s = (y - c_s - \tau_s) + r_s b_{s-1},
\]

where \(0 < \beta < 1\) and \(\sigma > 0\), and transversality condition

\[
\lim_{T \to \infty} D_{s,s+T}^{-1} b_{s+T} = 0.
\]

Here \(D_{s,s+j} = \prod_{i=1}^{j} r_{s+i}\) and \(r_s\) is the gross real interest rate factor from \(s - 1\) to \(s\) determined at date \(s - 1\).
• Given interest rate expectations $r^e_{t+i}(t)$ at time $t$, we can compute the consumption function

$$c_t = (1 + S_D(t))^{-1}(r_t b_{t-1} + y - \tau_t + S_N(t)),$$

where

$$S_N(t) \equiv \sum_{j=1}^{\infty} (D^e_{t,t+j}(t))^{-1}(y - \tau^e_{t+j}),$$

$$S_D(t) \equiv \sum_{j=1}^{\infty} \beta^{j/\sigma}(D^e_{t,t+j}(t))^{1/\sigma-1},$$

$$D^e_{t,t+s}(t) = r_{t+1} \prod_{i=2}^{s} r^e_{t+i}(t), \ s \geq 2 \ \text{and} \ D^e_{t,t+1}(t) = r_{t+1}. $$

• The market clearing condition is

$$c_t + g_t = y.$$
4.2 Balanced Budget for Government

- It is maintained that

\[
\begin{align*}
\tau_t &= g_t \text{ for all } t \\
b_{t-1} &= 0.
\end{align*}
\]

- Assume that at the beginning of \( t = 1 \), it is announced that

\[
\begin{align*}
g_t &= g_0 \text{ for } t < T_p \\
g_t &= g_1 (> g_0) \text{ for } t \geq T_p.
\end{align*}
\]

- The RE benchmark is

\[
\begin{align*}
r_t &= \beta^{-1} \text{ for } t < T_p - 1 \text{ and } t \geq T_p \\
r_{T_p-1} &= \beta^{-1} \left( \frac{y - \tau_1}{y - \tau_0} \right)^\sigma.
\end{align*}
\]
• Learning (i) agents forecast future taxes by computing per capita taxes from $g_t$ and (ii) must forecast $r_{t+i}^e(t)$.

• The forecasting model is $r_{t+i}^e(t) = r^e(t)$, so called steady-state learning (with constant gain). We calculate

$$r_{t+1} = \left[ \frac{1 - \beta^{1/\sigma}r^e(t)^{1/\sigma-1}}{\beta^{1/\sigma}(1 - (r^e(t))^{-1})} \right] \left(1 - \frac{(\tau_1 - \tau_0)}{(y - \tau_0)}(r^e(t))^{t+1-T_p}\right)$$

for $1 \leq t \leq T_p - 1$,

$$r_{t+1} = \left[ \frac{1 - \beta^{1/\sigma}r^e(t)^{1/\sigma-1}}{\beta^{1/\sigma}(1 - (r^e(t))^{-1})} \right]^\sigma$$

for $t \geq T_p$.

• For analytic results, assume $\sigma = 1$. We have:
Proposition 3 Assume that the economy is initially in a steady state and consider a permanent government spending and tax increase $\tau_1 > \tau_0$, which takes place in period $T_p$ and which is announced during period 1. Under learning, for all $\gamma > 0$ sufficiently small we have:

(a) Interest rate expectations satisfy $(1 - \gamma)^{-1} < r^e(t) < \beta^{-1}$ for all $t \geq 2$ and $\lim_{t \to \infty} r^e(t) = \beta^{-1}$.

(b) The actual temporary equilibrium interest rate $r_{t+1}$ satisfies:

(i) $r_{t+1} < \beta^{-1}$ for $t = 1, \ldots, T_p - 1$,

(ii) $r_{t+1} > \beta^{-1}$ for $t \geq T_p$ and $\lim_{t \to \infty} r_{t+1} = \beta^{-1}$.

- The two figures provide dynamics of $r_t$ under learning and under RE as well as expectations under learning.
$r_{t+1}$ under learning (solid line) and $r_{REE}$ under rational expectations (dashed line), balanced-budget case with the permanent policy change.
Interest rate expectations, balanced budget case with permanent policy change. The dashed line is the steady state value $\beta^{-1}$. 
Other cases studied in the paper:
- Temporary changes in spending.
- Learning with repeated policy implementations: agents’ behavior may converge to the RE with these anticipated policy changes.
- Debt Financing: cases of near Ricardian Equivalence can emerge.
- The Ramsey Model: more cyclical dynamics than under RE.